Trading loss against delay in Networked Control Systems

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Digitally Networked Dynamical Systems

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Feedback Control

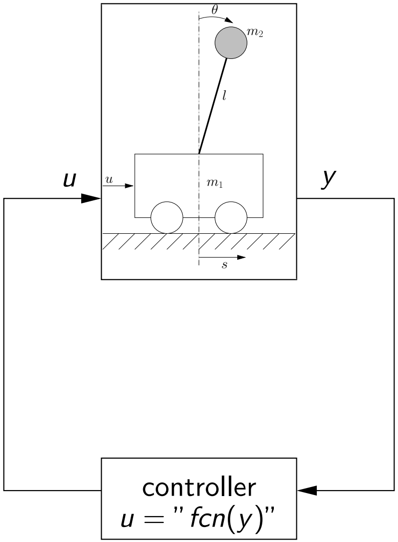
Show nice video

http://www.youtube.com/watch?v=cyN-CRNrb3E

http://www.youtube.com/watch?v=Ep2lNMic_fk

http://www.youtube.com/watch?v=B6vr1x6KDaY

Control System



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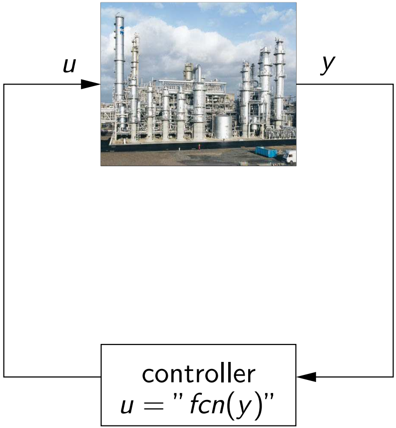

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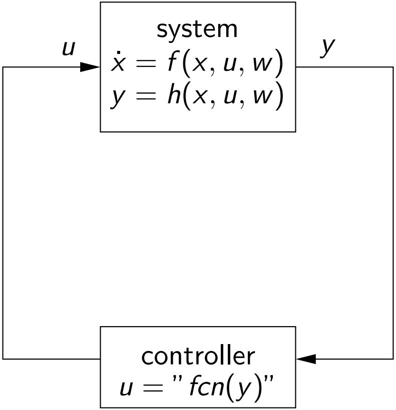
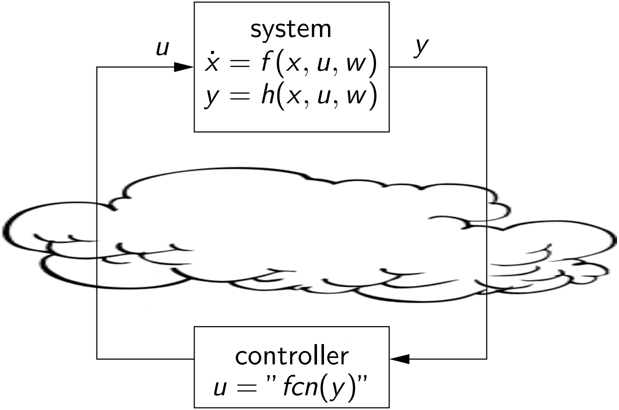
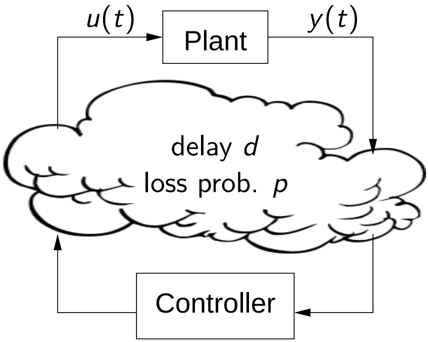


Networked Control System



Loss and Delay in Networked Control Systems



Loss and Delay

- In control literature: assumed to be fixed.
- In communication literature: depends on network usage.

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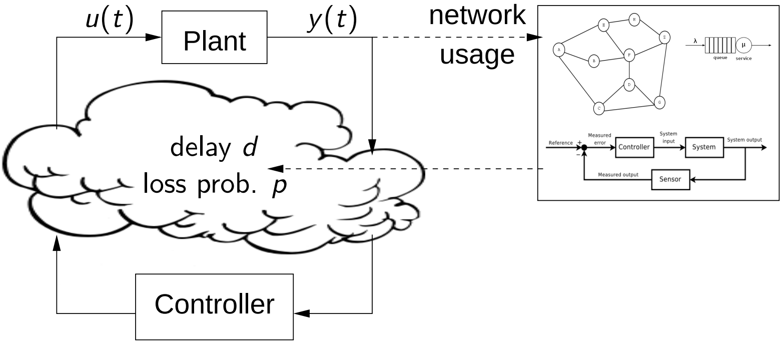

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Outline

1. Introduction
2. Problem Setup
3. Comparing Time-Triggered and Event-Based Control
4. Trading Loss Against Delay
5. Outlook: Scalar System
6. Summary

Problem Setup

N Systems

$$\dot{x}(t) = u(t) + w(t),$$

where $u(t)$ is the control input and $w(t)$ noise.

Impulsive input

$$u(t) = \sum_k -\delta(t - t_k)x(t),$$

each impulse resets the state $x(t)$ to the origin.

Performance Measure (Cost)

$$J = \limsup_{M \to \infty} \frac{1}{M} \int_0^M \mathrm{E}[x(t)^2]dt$$

Closed Loop System

$$t \neq t_k: \quad u(t) = 0,$$
$$\dot{x}(t) = w(t),$$
$$t = t_k: \quad x(t_k) = 0.$$

Performance Measure (Cost)

$$J = \limsup_{M \to \infty} \frac{1}{M} \int_0^M \mathrm{E}[x(t)^2] dt$$

Problem Setup

Closed Loop System

$$
\begin{aligned}
t \neq t_k : \quad & u(t) = 0, \\
& \dot{x}(t) = w(t), \\
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\end{aligned}
$$

When to reset? How to choose t_k?

Performance Measure (Cost)

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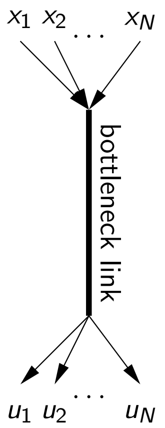
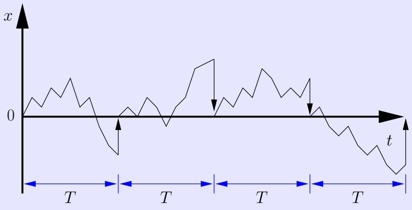


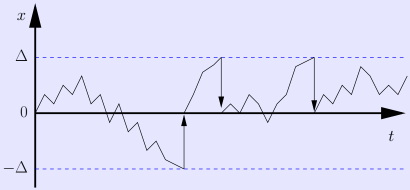
Comparing Time-Triggered and Event-Based Control

Time-Triggered Control



Event every T seconds.

Event-Based Control



Event whenever $|x| \geq \Delta$.

Åström 02, Rabi 09, Blind 11

$$J_{tt} = \frac{T_{tt}}{2} + \frac{T_{tt}p}{1-p} + d$$

Åström 02, Rabi 09, Blind 11

$$J_{eb} = \frac{T_{eb}}{6} + \frac{T_{eb}p}{1-p} + d,$$

where $T_{eb} = \Delta^2$

Comparing Time-Triggered and Event-Based Control

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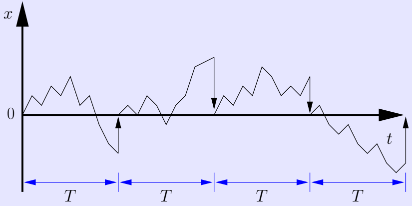

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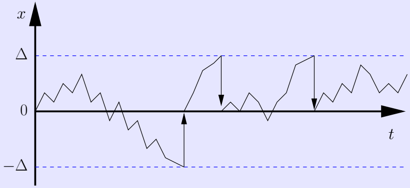

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Literature: Event-based control is superior for NCS because fewer events are necessary to get the same performance.

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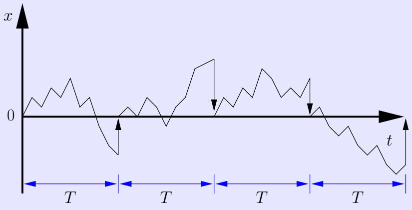

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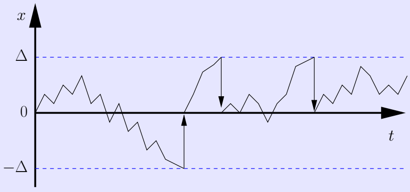

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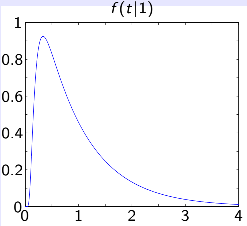
Literature: Event-based control is superior for NCS because fewer events are necessary to get the same performance.

The traffic pattern is different.

Interevent Time Distribution (Rabi 2009)

The Probability Density Function (PDF) $f(t|\Delta)$ of the interarrival times T_{eb} of event-based control is given as:

$$f(t|\Delta) = \Delta\sqrt{\frac{2}{\pi t^3}} \sum_{k=-\infty}^{\infty} (4k+1) e^{-\frac{(4k+1)^2\Delta^2}{2t}}.$$



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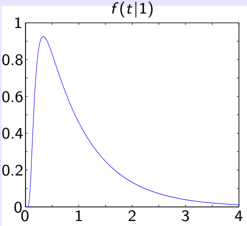

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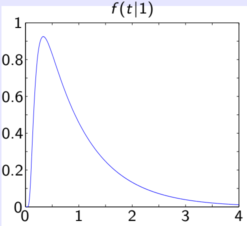

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- 'Nice' traffic.
- Possible to use 'classic' theory to analyze NCS.

Trading Loss Against Delay

Time-Triggered Control

$$\frac{J_{tt}}{\tau} = \frac{1}{2\rho} + \frac{p}{\rho(1-p)} + \frac{d}{\tau}.$$

Event-Based Control

$$\frac{J_{eb}}{\tau} = \frac{1}{6\rho} + \frac{p}{\rho(1-p)} + \frac{d}{\tau}.$$

TDMA

ALOHA

FDMA

Infinite queue

τ: packet duration.
$\rho := \tau/T$: load of one agent.
$\rho_\Sigma = N\rho$: offered network load.

Erlang's loss model

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- $d = \tau$
- $\rho_\Sigma \leq 1$

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ALOHA

- $p_u = 1 - e^{-2\rho_\Sigma}$,
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- $d_u = \tau$, $d_s = 1.5\tau$

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Infinite queue

- $p = 0$
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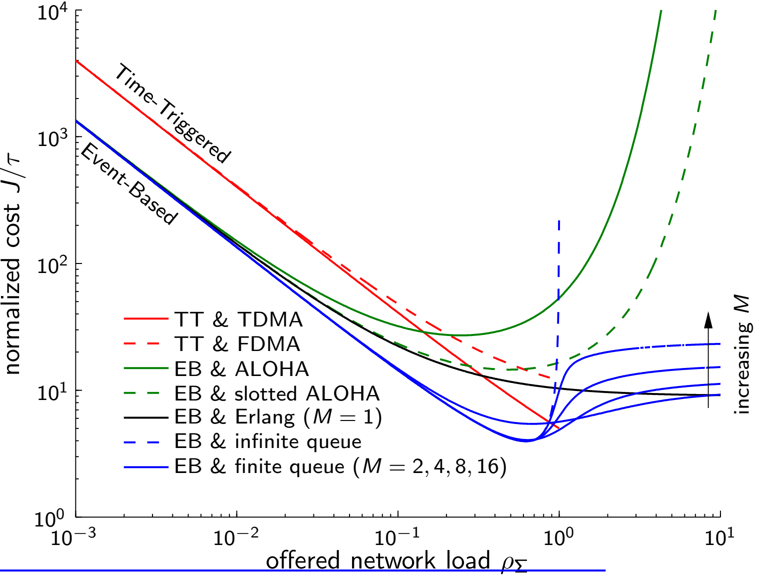
Erlang's loss model

- $p = \frac{\rho_\Sigma}{1+\rho_\Sigma}$
- $d = \tau$

Normalized Cost for $N = 8$ agents



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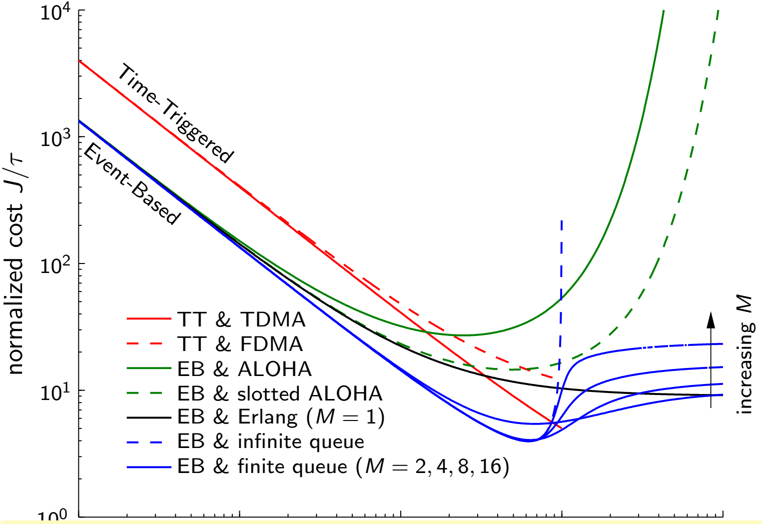

Performance depends on the control and communication strategy.

Outlook: Scalar System

Scalar System

$$\dot{x} = ax + u + w$$

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Theorem (Blind 2012)

Suppose, the scalar system is controlled by an event-based control scheme with bounds $\{\Delta_i\}$ and a packet loss probability p. Then the cost is

$$J = \frac{\sum_{i=0}^{\infty} p^i \int_0^{\Delta_i} \int_0^t x^2 e^{a(x^2-t^2)} dx dt}{\sum_{i=0}^{\infty} p^i \int_0^{\Delta_i} \int_0^t e^{a(x^2-t^2)} dx dt}.$$

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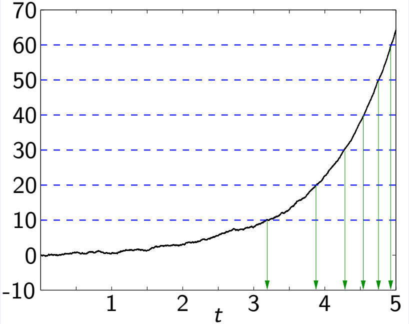
How to choose the bounds $\{\Delta_i\}$?

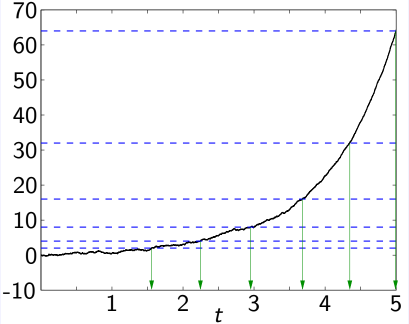
How to Choose the Bounds?

Additively Increasing



- Acceptable for control.
- Bad for communication?

Multiplicatively Increasing



- Very bad for control.
- Good for communication?

Summary and Outlook

Time-Triggered Control

Deterministic traffic

$$\frac{J_{tt}}{\tau} = \frac{1}{2\rho} + \frac{p}{\rho(1-p)} + \frac{d}{\tau}.$$

Event-Based Control

Poisson Traffic (for $N \to \infty$).

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Conclusion

Performance depends on the control and communication scheme.

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Outlook

- Scalar systems.
- Network instead of one bottleneck link.
- Send optimally.
- More realistic communication protocols.