Availability Estimation of Optical Network Links using Multilevel Bayesian Modeling

ITG KT 3.3 Workshop “Design, Operation and Automation of Open Transport Networks“

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Introduction

Availability

The probability that a continuously operating system that is undergoing repair after each failure is found in the “up” state at any random time in the future

True availability

$$a = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

Empirical Interval availability

$$a = \frac{\text{uptime}}{\text{uptime} + \text{downtime}}$$

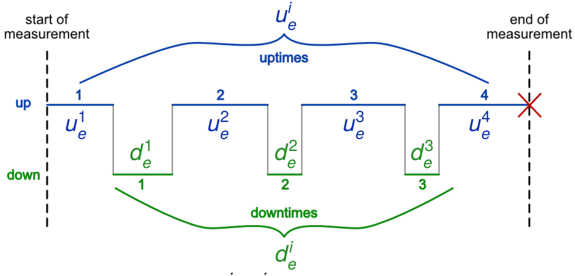
Estimated Availability

$$\hat{a} = ?$$

Find a better way to estimate network link availability.

Observed Data and Modeling

Data taken from observing a link $e \in E$



Total uptime

$$t_e^{\mathrm{u}} = \sum U_e = \sum_i u_e^i$$

Total downtime

$$t_e^{\mathrm{d}} = \sum D_e = \sum_i d_e^i$$

The data retrieved include $u_e^i, d_e^i \; \forall \; e \in E$.
We assume the following likelihood model

$$u_e^i \sim \text{Exponential}(\hat{\mu}_e^{\mathrm{f}}) \; \forall \; i \in |U_e'| \; \forall \; e \in E$$

$$d_e^i \sim \text{Exponential}(\hat{\mu}_e^{\mathrm{r}}) \sim \text{Exponential}((\hat{\mu}^{\mathrm{r}})) \; \forall \; i \in |D_e'| \; \forall \; e \in E$$

$$|U_e'| \sim \text{Poisson}(t_e^{\mathrm{u}} / \hat{\mu}_e^{\mathrm{f}}) \; \forall \; e \in E$$

$$\sum_e |D_e'| \sim \text{Poisson}\left(\frac{\sum_e t_e^{\mathrm{d}}}{\hat{\mu}^{\mathrm{r}}}\right)$$

Finding priors

Assume an Inverse-Gamma prior for the Exponential likelihood model:

$$\hat{\mu}_e^{\mathrm{f}} \sim \text{Inverse-Gamma}(10, \beta_e^{\mathrm{f}})\ \forall\ e \in E$$
$$\hat{\mu}^{\mathrm{r}} \sim \text{Inverse-Gamma}(2, 40)$$

Failures depend on the link length with a reciprocal relationship

$$\tilde{\mu}^{\mathrm{f}}(x; k, s) = k + \frac{s}{x}$$
$$k \sim \text{Gamma}(0.8, 9)$$
$$s \sim \text{Gamma}(2, 155)$$

We set the reciprocal relationship to be the average of the Inverse-Gamma distribution

$$\beta_e^{\mathrm{f}} = 9\tilde{\mu}^{\mathrm{f}}(l_e; k, s)\ \forall\ e \in E$$

Statistical model

$$
\begin{aligned}
u_e^i &\sim \text{Exponential}(\hat{\mu}_e^{\mathrm{f}})\ \forall\, i \in |U'_e|\ \forall\, e \in E \\
d_e^i &\sim \text{Exponential}(\hat{\mu}^{\mathrm{r}})\ \forall\, i \in |D'_e|\ \forall\, e \in E \\
\hat{\mu}_e^{\mathrm{f}} &\sim \text{Inverse-Gamma}(10, \beta_e^{\mathrm{f}})\ \forall\, e \in E \\
\hat{\mu}^{\mathrm{r}} &\sim \text{Inverse-Gamma}(2, 40) \\
\tilde{\mu}^{\mathrm{f}}(x; k, s) &= k + \frac{s}{x} \\
k &\sim \text{Gamma}(0.8, 9) \\
s &\sim \text{Gamma}(2, 155) \\
\beta_e^{\mathrm{f}} &= 9\tilde{\mu}^{\mathrm{f}}(l_e; k, s)\ \forall\, e \in E \\
|U'_e| &\sim \text{Poisson}\left(\frac{t_e^{\mathrm{u}}}{\hat{\mu}_e^{\mathrm{f}}}\right)\ \forall\, e \in E \\
\sum_e |D'_e| &\sim \text{Poisson}\left(\frac{\sum_e t_e^{\mathrm{d}}}{\hat{\mu}^{\mathrm{r}}}\right)
\end{aligned}
$$

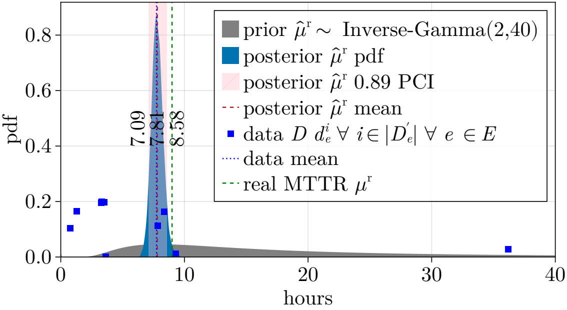
Prior Analysis & Inference for estimated MTTR

Estimated MTTR $\hat{\mu}^{\mathrm{r}}$ has a prior of Inverse-Gamma$(2, 40)$

- non negative
- discourages too low MTTR values
- does not exclude high MTTR values

After inference on the blue squared downtimes

- average of the posterior distribution is 7.81 (almost same as data mean)
- 89% percentile central interval (PCI) in $(7.09, 8.58)$



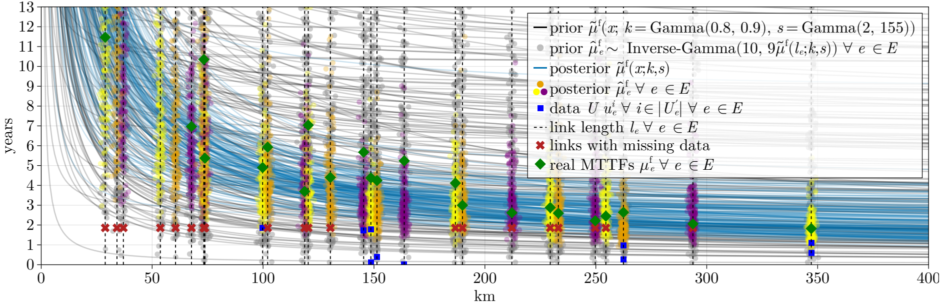
Prior Analysis & Inference for estimated MTTF

We have 2 priors because of multiple levels in the model

- $\tilde{\mu}^{\mathrm{f}}$ models the reciprocal relationship and the prios are the gray lines
- $\hat{\mu}^{\mathrm{f}}_{e}$ models the MTTF for each line and the priors are the gray scattered points

After inference on the blue squared uptimes datapoints and interrupted measurements

- the reciprocal relationship posterior signified with blue lines is very reasonable
- the link MTTF posterior signified with the colorful points are stricter and within range
- pure data point average cannot compete with the model estimations

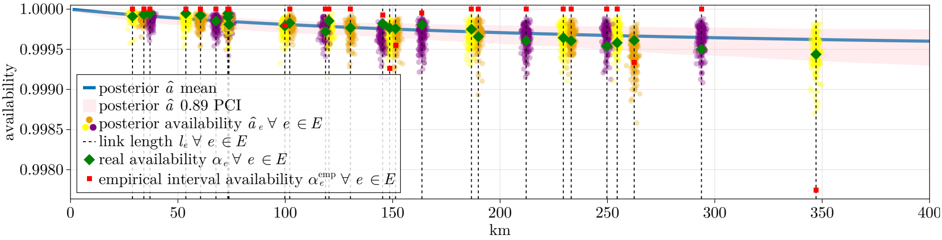


Estimated Availability Evaluation

Since we know the estimated MTTR $\hat{\mu}^{\mathrm{r}}$ and estimated MTTF $\hat{\mu}^{\mathrm{f}}_e$ we can calculate the estimated availability per link

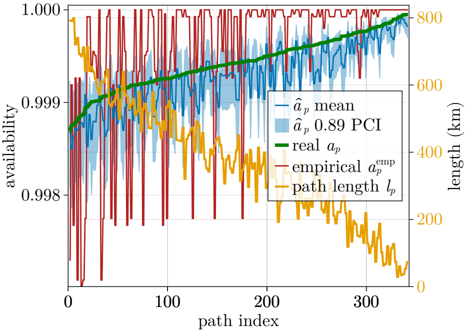
$$\hat{a}_e = \frac{\hat{\mu}^{\mathrm{f}}_e}{\hat{\mu}^{\mathrm{f}}_e + \hat{\mu}^{\mathrm{r}}}$$

- the availaility estimations per link include the real availability of the underlying stochastic process
- the empirical interval availability is mostly over- or underestimating



Comparison with Empirical Availability

We calculated the overall availability for all end-to-end paths by multiplying the availability estimations for each link.



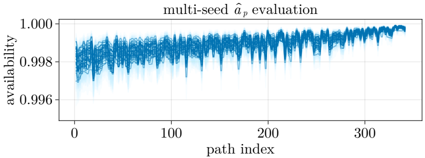
empirical path availability

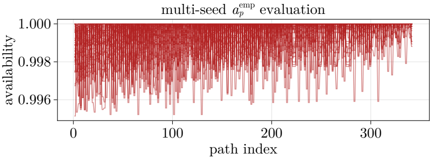
- spontaneous, erratic
- high variance

modeled path availability estimation

- lower error
- more consistent
- uncertainty representation

Given enough data the empirical and modeled estimated availabilities will converge





Conclusion

We built a Bayesian model to estimate the true availability for network links and paths

- uninformative priors
- successful inference

We evaluated it and compared it with the empirical availability

- model estimations are more accurate
- model estimations are more consistent
- model estimations do very good with scarce data
- model estimations provide uncertainty assesements

Future work

- incorporate this model for decision-making scenarios
- extend the model to include node availabilities and other QoS metrics