ABSTRACT

This paper deals with teletraffic theory as an important tool to solve problems of telecommunication engineering. It is shown that by thorough simplifications -- theoretical results can be made applicable to a handy use in practice.

As an example, the calculation of arbitrary one-stage connecting arrays and link systems are considered in this paper.

Furthermore, special importance is attached to the problem how to dimension economically telephone networks with alternate routing.

As final result, a table book could be edited which enables, for the first time, this complicated task to be handled with high accuracy by means of a few tables only.

1. INTRODUCTION

1.1 General

During the Sixth ITC in Munich the President, Professor Dr. A. Jensen /27/, Mr. R.J. Chapuis /12/, Mr. I.A. Newstead and Mr. I. Tange /45/, Mr. P.G. Wright /60/ and many other prominent participants were focusing upon the necessity of a close cooperation between CCITT and ITC regarding the increasing need of handy teletraffic outlines for field engineering. I should like to cite only one of these important voices, namely Dr. Jacobseus /26/ mentioning in his invited paper:

"...we should also direct our attention to some extent to the technical applications. We need to be reminded that our task is, together with the technicians and operations personnel of the manufacturers and administrations, to achieve better telecommunications plant, plant which is better matched to fulfill its functions for society and for the subscribers. To achieve this aim, theoretical, technical and purely practical advances must go hand in hand". So far Dr. Jacobseus.

The aim of my paper is to present some typical examples considering these important objectives being of assistance and help to field engineering problems. I should like to demonstrate the possibility to start from theoretical considerations achieving, nevertheless, handy solutions for practical engineering. However, the road to success, i.e. leading stepwise from a more interesting theory to methods of practical applicability and without losing significantly accuracy, can be long and hard. It desires also good contacts to manufacturers and administrations. Finally, one should achieve outlines for the considered problem which can be applied easily and correctly by field engineers without considering the entire theoretical background, perhaps even without knowing much about it.

1.2 The Problems to be Solved

Outlines should be developed for loss systems regarding the following problems:

a) Probability of loss of one stage arrays and link systems with offered traffic PCT 1 or PCT 2, respectively.

b) Variance V of overflow traffic R behind groups with k + n.

c) Dimensioning of groups with offered overflow (A<sub>n</sub>, V).

d) Economic partitioning of the traffic to alternate routes.

e) Tabulated methods to b), c) and d) for field engineering.

f) Direct application of a) to link systems.
2. THE PROBABILITY OF LOSS IN ONE STAGE ARRAYS AND LINK SYSTEMS

Remark:
The alternate routing outlines described in Chapters 3 and 4 assume offered random traffic only from an infinite number of sources (Poissonian input). Nevertheless, for reasons of completeness, also loss calculation methods for a finite number of source can be concisely mentioned. Regarding the overflow problem for finite source traffic see /53,54/. As to the calculation of groups with offered smooth traffic see /52,53,54,42/.

2.2 Trunk Groups behind Gradings with Limited Accessibility

2.2.1 Exact Calculation

The exact calculation of the probabilities of state in such groups, as well as of the loss probability, has been treated in many papers. Enormously large systems of linear equations generally to be evaluated numerically (except in case of Erlang's Ideal Gradings) do not permit the application of these exact methods in practice. (References see e.g. /9, 36/).

The number of interesting methods for an approximate loss calculation is therefore very high and still increasing. A survey can be found in /36/.

2.2.2 Grading Types

A large variety of different grading types is in use. Four typical grading types are shown in Fig. 1. Each one of these examples has g=9 selector groups (with e.g. 10..50 inlets each). The accessibility is critical to the number of outgoing trunks n=45. The mean interconnection number ("grading ratio") is M E-k/n = 2 for each of these grading examples.

Within PBX and for other connecting arrays with a comparatively small number of traffic sources (q ≤ 5..n) one applies Erlang's Bernoulli Formula (RF) /11/ for finite number of sources, tabulated e.g. in /48,63/.

Here we have a constant call rate α per idle source; this type of traffic is also called "Pure Chance Traffic of Type 1; PCT 1".

\[ B_n(A) = \frac{A^n}{n!} \frac{1}{\lambda h} \cdot \frac{1^A}{1^0} \]  

with
- \( \lambda \) = arrival rate
- \( h \) = mean holding time
- \( A \) = offered traffic
- \( n \) = number of trunks in the group,
interarrival time and holding time being negative exponentially distributed. These assumptions are realistic enough for telephone traffic as checked by many extensive measurements, e.g. /14,23,25/.

Within PBX and for other connecting arrays with a comparatively small number of traffic sources (q ≤ 5..n) one applies Erlang's Bernoulli Formula (RF) /11/ for finite number of sources, tabulated e.g. in /48,63/.

Here we have a constant call rate \( \alpha \) per idle source; this type of traffic is also called "Pure Chance Traffic of Type 2; PCT 2".

\[ \frac{q^n}{n!} \cdot \frac{1^q}{1^0} \frac{n}{q+q} = \frac{q^n}{n!} \cdot \frac{1^q}{1^0} \frac{n}{q+q} \]  

with
- \( q \) = number of traffic sources
- \( \alpha \) = call rate per idle source
- \( n \) = number of trunks in the group
- \( h \) = mean holding time (h=1)
- \( Y \) = carried traffic

Interarrival time per idle source and holding time being negative exponentially distributed.

Fig. 1: Typical Grading Types (n=45, k=10, g=9)

So-called "perfect" gradings with skipping (and sometimes also with slipping), e.g. Type 1 in Fig. 1, have comparatively small losses. Nevertheless, also simplified grading types yielding slightly higher losses /23/, e.g. Types 2, 3 and 4 in Fig. 1, can sometimes be economical because of the reduced manpower which is necessary for their performance or extension, respectively. It should be mentioned, however, that simplified gradings will be more loss sensitive in case of unbalanced offered traffic.

Here, the problem was to develop a loss formula for offered Poisson traffic (PCT 1) which was adaptable to all grading types and yielding losses close to the results of artificial traffic tests (and/or to measurements in real systems) from very low values of loss (≤ 1%/oo) to very high ones (≥ 50 %).

2.2.3 Approximate Loss Calculation

For offered PCT 1

One way to achieve a good approximate loss formula for a certain grading type is to apply the statistical equilibrium equations as used for "Erlang's Ideal Gradings". In this case one has to find out another (approximate) function \( c(x,k,n,A,\text{grading type}) \) for the state blocking probability in any state \( x \) which describes sufficiently accurate the special properties of the respective grading type /10,19,29,38/.
Investigations which lasted for many years were leading in 1961, however, also to another solution by means of the Modified Palm Jacobaeus Formula (MPJ)/23,34,38/. This loss formula yields reliable values of loss for "perfect" gradings (see Type 1 in Fig. 1) with skipping and sometimes also slipping, if a minimum grading ratio M=2 is fulfilled. It holds for an accessibility \( k \)

\[
E_{MPJ} = E_{n-k}(A_0) \frac{E_n}{1-E_n(A_0)}
\]

with \( Y = A_0[1-E_n(A_0)] \) being prescribed for the calculation of \( E_{MPJ} \)

and with

\[
A_{actual} = Y \frac{1-E_{MPJ}}{E_{MPJ}}
\]

The admissible traffic \( A_{adm} \) of the modified Palm Jacobaeus formulae was offered to simple fixed grading types (No. 2,3,4 in Fig. 1 and similar ones) has smaller values for the same parameters \( n,k,h \). For this purpose an adaptation formula was developed /21/. One obtains

\[
A_{adm} = A_{MPJ} - \Delta A
\]

With

\[
\Delta A = P \cdot \left( \frac{n}{k} - 1 \right)^2 \cdot \frac{n-k-2}{1-k-B} \cdot \frac{1-B}{1+k-B}
\]

For a (standardized) minimum grading ratio \( n=2 \) the fitting parameter \( P \) depends on the grading type only. It can be determined by one single series of informing traffic trials for the corresponding type. This parameter is e.g. \( P=0.3 \) for Standard Gradings /15/ (Type 2 in Fig. 1). Furthermore, \( P=1.1 \) holds for O’Dell Gradings (Type 3 in Fig. 1) and \( P=2.4 \) for AT&T Gradings (Type 4 in Fig. 1).

Formula (4) is valid for a boundary number of trunks \( n_b \). For numbers \( n_b > n \) the corresponding admissible traffic \( A(n_b) \) is calculated -- regarding the prescribed loss \( B -- as\)

\[
A(n_b) = A(n_b) - \frac{n_b}{n} \cdot \frac{1}{h} \cdot \left( \text{where } n_b = 10 \cdot k \right)
\]

The reliability of this (heuristic) adaptation has been checked and is proved by extensive artificial traffic trials. Fig. 2 demonstrates this accuracy for the four grading types in Fig. 1 as an example with \( n=45 \), \( k=10 \) and \( g=9 \).

This MPJ formula with \( P=0.3 \) is for instance part of the dimensioning outlines of the telephone administration in the FGÖ (Deutsche Bundespost). It is tabulated extensively for \( P=0 /40,61,52/ \) and \( P=0.3 /22,63/ \).

2.2.4 Approximate Loss Calculation for Offered PCT 2

Dimensioning formulae for field engineering have to be developed also for gradings with offered PCT 2, i.e., for random traffic from a finite number of traffic sources \( q \leq 5n \).

The following two approximate formulae yield results rather close to artificial traffic tests in case of "perfect gradings (cf. Type 1 in Fig. 1):"

a) The Bernoulli Interconnection Formula (BIF)

\[
B = \sum_{x=1}^{n} (q-x)p(x)\frac{x}{n}
\]

with

\[
p(x) = \frac{x-1}{1-\frac{(q)}{n}} \prod_{z=0}^{x-1} \left( 1 - \frac{z}{n} \right)
\]

and

\[
Y = \sum_{x=0}^{n} x \cdot p(x)
\]

\[
A = \alpha \cdot (q - Y) \cdot h
\]

\[
q = \text{number of sources}
\]

\[
\eta = \text{number of trunks}
\]

\[
\alpha = \text{traffic intensity per idle source}
\]

\[
h = \text{mean holding time (h=1)}
\]

\[
Y = \text{carried traffic}
\]

\[
A = \text{offered traffic}
\]

Fig. 2: Adaptation of the MPJ-Formula to Gradings of Various Type (Offered PCT 1: Constant Call Intensity per Multiple)
b) The Bernoulli Quotient Formula (BQF) \[ A_{PCT1} \]

\[ B_n(x_n^*, q) = \frac{B_{n-k}(x_n^*, q)}{B_n(x_n^*, q)} \]  

with

\[ B_{n-k}(x_n^*, q) = \frac{(q-1)^n}{\Gamma(n)} \cdot \frac{q^{-n}}{q-x_n^*} \]  

The calculation prescribes: \( Y, n, k \) and \( q \), hence

\( x_n^* \) by iteration.

The actual traffic offered is

\[ A_{actual} = \frac{Y}{1-B} \]

Tables for all practically arising numbers of sources \( q \), numbers of trunks \( n \), accessibility \( k \) and probabilities of loss \( B \) -- calculated by means of Equ. (6) or (7) -- would be very voluminous and unhandy.

This disadvantage can be avoided by a transformation which uses only available PCT1 loss tables, the EST Method; i.e. Finite Source Transformation /5/.

For any tabulated quadrupel \( (A_{PCT1}, B, k, n) \) the increase \( A_{q/n} \) of the offered traffic which is admissible for the actually concerned source to trunk ratio \( q/n \) can be read from the corresponding loss tables of this grading type and for offered PCT1, with

\[ A_{q/n} = \frac{0.77 \cdot A_{PCT1}}{1-B} \]  

\[ A_{adm}(\frac{q}{n}) = A_{PCT1} + A_{q/n} \]

\[ A_{PCT1} \] for considered grading type and tables.

For instance one considers a standard grading (cf. Type 2 in Fig. 1) with a fitting parameter \( F < 0.5 \). Fig. 3 shows the accuracy of the EST Method as well as the influence of \( q/n \) on the probability of loss.

2.3 Loss Calculation and Structures of Two-Sided Link Systems

2.3.1 Exact Calculation

Although only stage arrays with full or limited access will be of practical interest still for many years the use of link systems is making progress.

To a large extent, the exact calculation of state distributions and loss probabilities of link systems has been theoretically solved for two-sided link systems with preselection, group selection, point-to-point selection /32/.

The rank of the concerning linear equation systems to be evaluated is, however, in case of usual sizes of link systems beyond any computer capacity at the present time. Nevertheless, the check of approximate solutions by exact solutions is now in sight, for at least small systems.

2.3.2 Approximate Loss Calculation

Field engineers need reliable and handy approximate loss calculation methods. Furthermore, outlines for crosspoint economic structures are desired. These problems have been investigated among others by the Stuttgart University Teletraffic Team since more than ten years, e.g. /30, 31, 37/. An analysis of calculation methods can be found in /30, 32/ containing also a large number of references.

Reliable approximate loss formulae should be applicable for losses between \( \leq 10^{-6} \) and \( \leq 50 \% \), in particular for point-to-group hunting with regard to systems with high usage groups and alternate routing. As continued development of former methods /35, 37/ two closely related new methods for loss calculation are therefore presented -- together with further results -- in another paper of this ITC /7/. These methods meet the above mentioned demands.

Besides the case of group selection and offered PCT1 dealt with in /7/, this loss calculation method can analogously be applied also to systems with traffic concentration in the first and/or further stages, furthermore to link systems with offered PCT2 (finite number of sources). An example for the demonstration of accuracy is given in Fig. 4.

The special influence to the loss of a so-called "internal traffic" running twice through a link system has also been investigated in detail and is published at this ITC /6/.

![Fig. 3: Probability of loss B vs. the offered traffic per trunk A/n for n=30, k=10 and varying source to trunk ratio q/n for a standard grading. (Simulation with 95% confidence interval)](521/4)
3. TELEPHONE NETWORKS WITH ALTERNATE ROUTING

3.1 General Problems Regarding Alternate Routing

Each telephone administration needs sufficiently accurate and easily applicable outlines for the economic dimensioning of trunk groups in dialing networks with alternate routing (local, nationwide or international networks, respectively).

The problems to be solved can be seen from Fig. 6A and 6B.

Fig. 6A: Tandem exchange with alternate routing

The exchange of Fig. 6A is drawn more schematically in Fig. 6B with regard to the various single and multiple overflows as well as to the directly offered traffic.

Fig. 6 is demonstrating that the outlines should have the following features:

- Dimensioning of trunk groups for prescribed probability of loss in case of offered random traffic and with full or limited accessibility (see Chapter 2).

- Outgoing trunk groups of a tandem exchange with alternate routing to which overflowing peaked traffic rests are offered have also to be dimensioned correctly for prescribed probability of loss or overflow, respectively. Suitable calculation methods are dealt with in Section 3.3.

- The traffic flowing through a tandem exchange with alternate routing have to be partitioned economically to the various alternatively hunted outgoing groups. This problem is treated in Section 3.4. The expression "economic" means that simple and fast dimensioning methods should yield a reasonably close approach to a theoretical cost minimum for all outgoing groups as a whole.

The developed methods as a whole will be summarized in Chapter 8.
3.2 Conditions for Practical Application

Any alternate routing dimensioning outline should take into account that in many countries the use of public DDD networks (besides eventually local alternate routing) has a very wide extent; e.g. in the FRG the administrative personnel has to perform traffic measurements and a new dimensioning of DDD trunk groups once or twice a year for about 6000 trunk groups outgoing from tandem toll exchanges.

The following conditions for applicable outlines result from those facts:

1. Variance to mean $V$ of overflowing traffic $R$ only from tables - if at all!

2. Only from tables:
   a) group size in case of offered overflow $(A_{ov})$,
   b) standardised cost ratios final/high usage trunks
   c) economic partitioning of the traffic to high usage groups and final group

3. No iteration procedures to be performed by planning engineers.

These conditions were strictly observed for the dimensioning outlines to be developed.

3.3 Dimensioning of Trunk Groups with Offered Overflow Traffic

As it is known, the ENT Method by Wilkinson and Riordan is used for groups with offered overflow traffic and full accessibility /59/.

For trunk groups with limited access the exact variance calculation is very complicated /78, 80, 51, 85/. A good and reliable approximate variance formula for limited accessibility whose structure is very similar to the exact one of J. Riordan (for full access) has, however, been found /20, 36/.

Equations (9), (10) and (11) show the concerning formulae for the variance $V$ of an overflowing traffic rest $R$

$$\text{Full Access (k:n) ENT Method: } \text{V} = R \cdot (1 + p \cdot R)$$

$$\text{Limited Access (k:n) RDA Method: } \text{V} = R \cdot (1 + p \cdot R \cdot \frac{R}{A_0})$$

where in any case

$$p = B \cdot (k + 1 - A_0 \cdot (1-B)) - 1$$

with

$$A_0 = A \text{ for full accessibility}$$

and

$$A_0 = f(k, B) \text{ and } B = f(A, p, k, \text{grading type})$$

in case of limited access.

As one can see, the function $p$ in Eq. (11) is only dependent on $k$ and $B$, respectively. Therefore, the above variance formula (10) is true for the overflowing traffic in all grading types with offered PCT1 because the pair $(B, k)$ implies already the relevant data of a certain grading type.

Fig. 7: Principle of the ENT Method and RDA Method

Fig. 7 shows the wellknown idea of R1. Wilkinson, i.e. how to calculate the loss of trunk groups to which overflow traffic is offered:

The actual overflowing traffic rest $\hat{R}_1 = \sum R_{1i}$ and its corresponding variance $\hat{V}_1 = \sum V_{1i}$ offered to a secondary group (left hand side) is generated as overflow of one fictitious substitute primary group (right hand side of Fig. 7) by means of an offered Equivalent Random Traffic $A^*_n$ and $n_\ast$trans. The quantities $A^*_n$ and $n_\ast$ have to be determined iteratively or by means of Rapp's approximate formula /45/.

In case of a secondary grading one needs also a substitute primary grading, whereas a further parameter $k^\ast$ has to be determined suitably /20, 36, 62/.

The loss probability of the total group with $n_\ast + P_{2i} k_\ast + k$ (on the right hand side) can now be calculated for any type of grading with high accuracy (cf. Section 2.23 and 2.1) as well as for full access.

Thus, one finds the overflowing traffic $R_2$ of the total substitute group and from $R_2$ one gets the probability of loss of the actual secondary group as

$$B_2 = \frac{R_2}{R_1}$$

The traffic $R_1$ is -- as offered traffic -- also called $A_{ov}$.

This method is both elegant and easy to understand, however, rather time consuming as soon as a large number of groups has to be dimensioned.

In order to save calculation time, it has been investigated whether an average, uniform "standardized" value $R = \hat{R}$ could be applied. As a result, the Central Telecommunications Department (PTZ) of the Deutsche Bundespost found that for all national DDD tandem exchanges in the public telephone network a uniform value $R = 1.5 \ldots 2.0$ would be accurate enough. Using only one parameter (e.g. $R = 2.0$) the number of necessary tables is significantly reduced. Furthermore, it saves any individual variance calculation by planning engineers!

Thus one obtains the simplest dimensioning procedure for groups with offered overflow traffic.
3.4 The Economic Partitioning of the Traffic Offered to Alternately Hunted Routes

The problem now to economically partition the traffic to the various high usage groups and at least to a final group regarding the different trunk costs per route is well known. I would like to refer only to the brilliant and extensive investigations of Y. Rapp (1954), C.W. Pratt (1954), R. Schehrer (1957) and many other scientists, e.g., F.5, 57, 87.

In each case a cost function has to be set up. Fig. 8 shows an example:

![Traffic flow in a tandem exchange](image)

Fig. 8: Traffic flow in a tandem exchange

The cost function holds

\[ c_{total} = c_{11} \cdot T_{11}(A_{11}, k_{11}, c_1, \Delta Y_1) + c_{12} \cdot n_{12}(A_{12}, k_{12}, c_2, k_3, n_2, c_3, c_5, g, \Delta Y_2, \Delta Y_3, \Delta Y_{11}) + c_{22} \cdot n_{22}(A_{22}, k_{22}, k_3, n_2, c_3, c_5, g, \Delta Y_2, \Delta Y_3, \Delta Y_{11}) + c_{33} \cdot n_{33}(A_{33}, k_{33}, k_3, n_3, c_3, c_5, g, \Delta Y_2, \Delta Y_3, \Delta Y_{11}) \]

where \( \Delta Y_2, \Delta Y_3 \) are the marginal occupancies of the groups No. 2 and 3 and \( \Delta Y_{11} \) is the marginal capacity of the final route.

One recognizes the mutual dependencies between all trunk groups being sequentially hunted.

Only the partial differentiation with respect to all free parameters, i.e., number \( n_{11} \) of trunks per group, can yield the strictly exact economical optimum, that is to say the minimum costs for all outgoing routes as a whole.

However, because of the mutual influence of each group to the other ones which are hunting in series, this could only be done by means of much too complicated iteration procedures. This leads (in particular for the case of multiple overflow) to further unavoidable simplifications for practical use.

1) One dimensions all high usage groups of second, third etc. choice uniformly for an overflow of B=20 % (in any case high occupancy and economy).

2) The marginal capacity \( \Delta Y_{11} \) of the final route is assumed to be only \( \Delta Y_{11} = f(k_{11}, B_{11}) \neq f(B_{11}) \).

3) Basing on 1) and 2) one obtains a simple formula for the cost ratio \( P \) in case of single, double, triple overflow by partial derivation of the cost function; see Fig. 9 (cf. annex in /22/).

![Network with triple overflow](image)

Fig. 9: Network with triple overflow

Partial derivation of the cost function yields for triple overflow a cost ratio

\[ P_{III} = \frac{1 - B_{II}}{f_{III}(1 - B_{II})} + \frac{B_{II}}{f_{III}(1 - B_{II})} + \frac{B_{III}}{f_{III}(1 - B_{II})} \]

from a table

(overflow \( B_{III} = B_{II} = 20 % = \text{const.} \)).

For double overflow \( B_{II} = 1.0 \) \( P_{II} = \frac{0.8}{f_{II}(1 - B_{II})} \).

For single overflow \( B_{III} = 1.0 \) \( P_{III} = \frac{0.8}{f_{III}(1 - B_{II})} \).

As it is known, the marginal occupancy of the considered first choice group has to be

\[ \Delta Y_{11} = \frac{\Delta Y_{11}}{P} \]

where

\( P = f_{I} \) or \( f_{II} \) or \( f_{III} \), respectively. Therewith \( n_{11} \) can be determined. Details of the derivation can be drawn from the Annex in /22/.

4. THE DEVELOPED PLANNING METHOD

4.1 General

Basing on the considerations of Sections 3.2, 3.3 and 3.4 a Table Book /22/ was calculated which allows a very fast and economical dimensioning of the outgoing trunk groups in any tandem exchange with alternate routing. An extremely high number of test calculations has proved that the achieved network costs are very close (approx. 1.5%) to the theoretical minimum. Hereby the unavoidable inaccuracies of forecasted traffic values, standardized cost ratios etc. are not yet regarded.
4.2 The Procedure of Dimensioning

In any case the high usage groups of first order are dimensioned first, then the ones of second, third order etc. and at least the final group, respectively.

The accessibility of high usage groups of first, second ... order and the final group is set up before dimensioning the number of trunks and selectors (to be possibly changed again if the sum of available outlets of the respective switching stage is too small).

In this regard one has to notice that the subsequently hunted high usage groups of first order, furthermore possible high usage groups of second, third ... order and the final group as a whole have the characteristic features of one grading.

Therefore, the smallest necessary total number of outgoing trunks for a given overall grade of service is obtained (similar to an ordinary grading which is sequentially hunted from fixed home position), if the number of interconnected outlets increases from the first to the last hunting position.

Consequently, \( Z_{m'} = \sum_{i=1}^{m} \frac{e_{i}}{m_{i}} \) has to be realized for the grading ratios of the individual groups (whereby \( m \) is the number of grading (partial access) groups).

The use of the three main types of tables is explained by the example given in Fig.10.

![Fig. 10: Network with double overflow](image)

a) Dimensioning of High Usage Groups of First Order (First Choice Groups):

The number of trunks \( n_{i} \) and the corresponding overflow traffic \( R_{i} \) of the respective high usage group of first order can be read from the Table Fig. 11.

By means of the following data

- \( A_{i} \) offered random traffic to the respective group No. 11
- \( k_{i} \) accessibility of this group No. 11
- \( R_{i} \) accessibility of the final group
- \( P \) cost ratio regarding the costs per trunk in the group No. 11, as well as the trunk costs in the subsequently hunted groups (cf. Section 3.4 and the Annex in /22/)

one obtains from this Table in Fig. 11

- \( n_{i} \) the number of trunks of the group No. 11
- \( R_{i} \) the overflowing traffic rest behind this group No. 11

<table>
<thead>
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<th>( k ) of the final group</th>
<th>Cost Ratio P</th>
<th>( k_{i} = 1 )</th>
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<td>( \leq 15 )</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>( &gt; 26 )</td>
<td>( n_{i} R_{i} )</td>
<td>( P )</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>3.0</td>
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</tbody>
</table>

Fig. 11:
Table for the determination of the number of trunks \( n_{i} \) and the overflowing traffic \( R_{i} \) for high usage groups of first order as a function of the offered random traffic \( A_{i} \), the cost ratio \( P \) and the accessibilities \( k_{i} \) of the high usage group of first order and \( k_{\text{fin}} \) of the final group.

b) Dimensioning of High Usage Groups of Second, Third ... Order:

The offered traffic \( A_{o, 21} \) (Fig.10) includes all offered overflow traffic rests \( R_{i} \) and the directly offered random traffic \( A_{21} \). These groups are dimensioned for a uniform probability of overflow \( Z = 0.20 \% \) and for the average (constant) variance-to-mean ratio \( Z = 2.0 \). The number of trunks \( n_{21} \) for \( A_{o, 21} \) and the given accessibility \( k_{21} \) can be obtained by the Table Fig. 12.

![Fig. 12: Table For the determination of the number of trunks n for high usage groups of second, third order etc. as a function of the offered overflow traffic A, the accessibility k and the probability of overflow B](image)

- \( B = 20\% \)
- \( Z = 2.0 \%
- \( n \)
- \( k \)
- \( A \)
- \( B \)

<table>
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<th>( 15 )</th>
<th>( k )</th>
<th>( n )</th>
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<td>28.8</td>
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<td>28.8</td>
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</table>

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Dimensioning of the Final Group:

In order to dimension final groups for a prescribed probability of loss, the Table in Fig. 13 is used.

<table>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>18.7</td>
<td>21.9</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>19.4</td>
<td>22.7</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>20</td>
<td>23.5</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>20.7</td>
<td>24.8</td>
<td>25</td>
<td>28</td>
</tr>
</tbody>
</table>

Fig. 13: Table for the determination of the number of trunks n for final groups as a function of the offered overflow traffic A, the accessibility k and the probability of loss B

8.4 Two Auxiliary Tables

One table allows the determination of the random traffic which is offered to a first high usage group (A_random = \( f(Y_{11}, \text{measured} \times \text{K}_{11}^* \)).

Another table allows the same procedure for high usage groups of second, third, etc., order to which overflowing traffic (B = 2.0 assumed) is offered.

8.3 Example

Let us regard a system with offered traffic, accessibilities and cost ratios, respectively, as in Fig. 14.

\[
A_{11} = 34 \text{ Erl} \quad A_{12} = 31\text{ Erl} \quad A_{21} = 295\text{ Erl} \quad A_{22} = 14\text{ Erl}
\]

\[
R_{\text{11}} = 2 \quad A_{\text{ov2}} = 10 \quad P_{\text{ov2}} = 0.2 \cdot A_{\text{ov2}}
\]

From Equation (14) one gets the cost ratios \( P_{\text{11}} \):

\[
P_{\text{11}1} = 2 \cdot 0.771 = 1.54 = 1.6
\]

\[
P_{\text{11}2} = 2.5 \cdot 0.771 = 1.93 = 2.0
\]

From the table in Fig. 11 one obtains

\[
n_{11} = 42 \quad n_{12} = 41
\]

\[
R_{11} = 3.40 \text{ Erl} \quad R_{12} = 2.29 \text{ Erl}
\]

Therewith

\[
A_{\text{ov},2} = R_{11} \cdot A_{\text{22}} + A_{\text{ov2}} \cdot R_{\text{22}} = 25.2 \text{ Erl}
\]

is the traffic offered to the second choice group.

From the table in Fig. 12: \( n_2 = 27 \).

The traffic offered to the final group is

\[
A_{\text{ov,fin}} = R_{2} \cdot A_{\text{fin,dir}} = (5.0 \times 14.0) \text{ Erl} = 19 \text{ Erl}
\]

From this follows by means of the table in Fig. 13

\[n_{\text{fin}} = 38\]

5. ON THE APPLICATION TO LINK SYSTEMS WITH
ALTERNATE ROUTING (POINT TO GROUP SELECTION MODE)

5.1 Extension of the Method to Link Systems

Up to now, the alternate routing problem has been discussed only with respect to one stage group selection arrays having constant accessibilities per outgoing group.

The same methods can also be applied to link systems by means of the same tables.

5.2 Link Systems with Full Accessibility

Link systems with exact full access according to Clos /15/ et al. furthermore "wide" group selection link systems with "practically" full access (cf. /7/) are not problematic.

The outgoing groups are calculated for offered random traffic as well as for offered overflow traffic like behind a one stage array with full accessibility.

5.3 Link Systems with Limited Accessibilities

Alternate routing with full accessible groups only, ie. not in any case the most favourable way. Full access to all trunk groups saves trunk but it leads to a comparatively small capacity reserve of the network as a whole. In case of a cable breakdown or other reasons of overload a highly used network with full access groups only has the smallest capacity reserve and could yield a high increase of loss /1,39/.

Therefore, link systems with limited access and a smaller trunk occupancy, but with a lower increase of loss in case of overload, could also be considered.

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The difficulty to dimension those link systems is due to the well known fact that the effective (limited) accessibility /7/ per outgoing group depends on the total traffic carried by the link system and the group size itself /17,18/. Consequently two ways of dimensioning are possible:

a) Iteration procedures regarding -- for prescribed traffic and loss -- the mutual dependency between accessibility and group size.

b) The design of smaller link system modules having practically full access to its N_i outlets. These N_i outlets are divided up to the outgoing groups and facilitate limited but constant accessibilities k_i to each group No. i, where \( \sum k_i = N_i \).

The outlets k_i per group No. i of such LCA-modules have now to be suitably graded to the respective number n_i of outgoing trunks (interconnection number N > 2). Such trunk groups could be dimensioned like one stage gradings. The tables according to Chapter 4 can be applied.

6. CONCLUSION

The methods described above prove that the results of teletraffic theory can, in many cases, be simplified so far that useful and practical methods are achieved for the daily work of telephone engineers without significant loss of accuracy.

7. ACKNOWLEDGEMENT

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