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Abstract—The currently emerging 802.16e (WiMAX) and 3GPP Long Term Evolution (LTE) cellular systems are based on Orthogonal Frequency Division Multiple Access (OFDMA). OFDMA suffers from heavy inter-cell interference if neighboring base stations use the same frequency range. One possible approach to solve this issue is the application of beamforming antennas in combination with interference coordination (IFCO) mechanisms between base stations. In this paper, we trace the problem of IFCO back to the graph coloring problem and investigate the achievable resource utilization of the interference coordinated system. We develop a heuristic that allows the combination of arbitrary scheduling algorithms with the IFCO mechanism. This allows an efficient utilization of the radio system’s frequency resources while still obeying scheduling constraints, such as Quality of Service requirements. Finally, we study the tradeoff between fairness and the total system throughput.

Index Terms—OFDMA, 802.16e, WiMAX, 3GPP LTE, interference coordination, beamforming, graph coloring, scheduling

I. INTRODUCTION

Several currently emerging standards for broadband cellular communication are based on Orthogonal Frequency Division Multiple Access (OFDMA). In particular, 802.16e (mobile WiMAX) and future 3GPP Long Term Evolution (3GPP LTE) cellular systems will offer high-speed packet switched services for a variety of applications. In OFDMA, the different users are multiplexed in time and frequency based on the underlying OFDM system, which basically corresponds to a combination of Frequency and Time Division Multiple Access (FDMA and TDMA). A major problem in FDMA/TDMA systems is the inter-cell interference that neighboring cells create when using the same frequency band. This may lead to severe performance degradation or connection loss especially in the border areas of cells.

Classical FDMA/TDMA systems, such as GSM, solve this problem by avoiding the reuse of the same frequency range in neighboring cells. This reduces the utilization of scarce frequency resources. Instead, it is desirable to be able to reuse the full frequency spectrum in every cell and mitigate the inter-cellular interference by other means. One promising approach is interference coordination (IFCO), where neighboring base stations coordinate their transmissions in order to minimize interference. This is particularly effective when combined with beamforming antennas, which additionally allows the exploitation of space-division multiplexing (SDM) and thus the transmission to spatially separated terminals on the same frequency/time resource.

IFCO has been an active research area in multi-hop and mobile ad hoc networking. In [1], the authors consider the possibility of beamforming in a multi-hop wireless network and study a MAC protocol which is capable of blocking the transmissions of the strongest interferers. In [2], the authors coordinate broadcasts in a multi-hop wireless network by means of a sequential graph coloring heuristic. In [3], the coordination of transmissions in a wireless ad-hoc network is considered. The interference conditions are evaluated by an omnipotent central entity with full system state information, which is able to schedule the data transmissions of the individual nodes on the MAC-frame level. This is done based on a conflict graph, which represents critical interference relations in-between the network nodes. The problem was traced back to the graph coloring problem for example in [4]. In [5], the throughput capacity of a wireless multi-hop network was calculated with the help of a very similar schedule graph, which is derived from physical layer properties of the network.

In the area of cellular networks, IFCO has gained little attention so far and only recently became an active research topic in the course of 802.16e and 3GPP LTE (e.g., [6]). Earlier references include [7] and [8], where the authors focus on a flow-level analysis of the possible capacity gains with inter-cellular coordination and a static resource assignment policy. In [9], we introduced the concept of an interference graph in cellular networks. The semantics of the interference graph are similar to those of the above mentioned conflict graph. However, compared to ad-hoc networks with actively transmitting nodes, only the base stations are transmitting when considering the downlink direction. This leads to a variation of the original problem. In [9], we used a simple but efficient heuristic in order to solve the resource assignment problem in combination with the interference graph, which requires an omniscient device will full system knowledge. Even though this is not feasible in a real system, it provides valuable insight on the impact of important system properties and eventually yields an upper bound for the achievable performance.

In this paper, we extend our research from [9] and first study the basic properties of the interference graph. We then solve the resource assignment problem by tracing it back to the graph-coloring problem. By using a tabu search algorithm, we obtain near-optimal colorings of the interference graph,
which allows us to draw conclusions on the resource utilization in the system. Subsequently, we study the impact of system constraints and scheduling on the resource assignment process. Finally, we present a heuristic that allows to trade off scheduling and fairness requirements with the total throughput.

This paper is structured as follows. Section II introduces the cellular scenario. Section III defines the interference graph and studies its properties. Subsequently, section IV studies the pure graph coloring problem, and section V the system imposed constraints and fairness issues. Finally, section VI concludes the paper.

II. SYSTEM MODEL

A. Simulation Scenario

Throughout our paper, we consider a hexagonal cell layout comprising 19 base stations at a distance of $d_{BS} = 1400$ m with wrap-around. This is shown in Fig. 1. The wrap-around property infinitely repeats the 19 base stations in the xy-plane. Therefore, there is no distinct center cell and all cells are equal. This leads to a completely symmetric and balanced scenario. Every base station has three $120^\circ$ cell sectors, where each sector is served by one transceiver. The transceivers are equipped with linear array beamforming antennas with four antenna elements and gain patterns according to [9]. They can be steered towards each terminal with an accuracy of $1^\circ$ degree, and all terminals can be tracked ideally. We perform Monte-Carlo simulations, where $N$ mobile terminals per cell sector are placed at different positions for each drop.

B. Overview of transmission system

We consider an 802.16e-system [10] with a total available system bandwidth of 10 MHz and a MAC-frame length of 5 ms. This results in a total number of 49 OFDM-symbols per MAC-frame and 768 data subcarriers per OFDM-symbol. All cells were assumed to be synchronized on a frame level. Each MAC-frame is subdivided into an uplink and a downlink subframe. Both subframes are further divided into zones, allowing for different operational modes. In this paper, we focus on the Adaptive Modulation and Coding (AMC) zone in the downlink subframe. In particular, we consider the AMC 2x3 mode, which defines subchannels of 16 data subcarriers by 3 OFDM-symbols. This is illustrated in the left part of Fig. 2. A subchannel corresponds to the resource assignment granularity for a particular mobile terminal. The AMC zone can therefore be abstracted by the two-dimensional resource field shown in the right part of Fig. 2, the resources of which need to be assigned to transmissions towards the mobile terminals.

III. INTERFERENCE COORDINATION AND RESOURCE ASSIGNMENT

A. General procedure

Each sector has its own set of resources (see Fig. 2), which need to be assigned to the terminals in every MAC-frame. In order to realize the coordination of cell sectors, we divide the scheduling process into two parts:

1) Creation of an interference graph: In this step, a graph is created based on the interference relations in-between all terminals. Its vertices represent the mobile terminals, and its edges represent critical interference relations in-between the terminals. In particular, terminals which are connected must not be served using the same set of resources. Figure 3 shows an example of an interference graph.

![Figure 1. Hexagonal cell layout with wrap-around](image-url)
As already mentioned, this interference graph is similar to the conflict graph used in [3] regarding its semantics. However, it is constructed differently in our case, since terminals are only receiving but not transmitting any data.

2) Resource assignment: In this second step, resources are assigned to the different terminals while taking into account the constraints of the interference graph. In section IV, the resource assignment problem is treated by means of graph coloring algorithms. In section V, we additionally take into account constraints of the actual 802.16e transmission system and present a heuristic scheduling algorithm.

In the following two subsections, we will first describe the construction of the interference graph and study its basic properties.

B. Construction of the Interference Graph

In this section, we briefly review the creation of the interference graph as we introduced it in [9]. We construct the interference graph by evaluating the interference that a transmission to mobile \( m_l \) in sector \( i \) would cause to mobile \( m_k \) in sector \( j \), where \( i \neq j \):

\[
I_{kl} = p_{ik} G_i(l, k) P_l ,
\]

where \( P_l \) is the transmission power of transceiver \( i \) towards terminal \( m_l \). For each terminal \( m_k \), we collect all interference relations in the set \( W_k \):

\[
W_k = \{ I_{kl}, \forall l \neq k, |c_l - c_k| \leq d_{ic} \}.
\]

We then keep removing the largest interferer from \( W_k \) until the worst-case SIR for terminal \( m_k \) rises above a given desired SIR threshold \( D_S \):

\[
S_k = \frac{\sum_{I_{kl} \in W_k} I_{kl}}{I_{kl}} \geq D_S .
\]

The edges \( e_{kl} \) of the interference graph then follow as:

\[
e_{kl} = \begin{cases} 
0 & \text{if } I_{kl} \in W_k \land I_{lk} \in W_l \\
1 & \text{otherwise}
\end{cases}.
\]

Equation (5) sets an interference relation \( e_{kl} \) if terminal \( m_k \) causes interference to terminal \( m_l \), or vice versa. This is necessary since in both cases the usage of the same set of resources must be avoided. This results in a non-directional interference graph, i.e., \( E \) is symmetric.

Finally, all mobile terminals within a cell sector must be assigned disjoint resources. Hence, \( e_{kl} = 1 \) if \( m_k \) and \( m_l \) beamforming antenna towards terminal \( m_k \) when the array is directed towards terminal \( m_l \). \( e_{kl} \in \{0,1\} \) are the elements of the interference graph’s adjacency matrix \( E \), indicating an interference relation between terminals \( m_k \) and \( m_l \) if \( e_{kl} = 1 \).

In a first step, we calculate the interference \( I_{kl} \) which a transmission to mobile \( m_l \) in sector \( i \) would cause to mobile \( m_k \) in sector \( j \), where \( i \neq j \):

\[
I_{kl} = p_{ik} G_i(l, k) P_l ,
\]

As in [3], \( G_i(l, k) \) is the path loss from transceiver \( l \) to terminal \( k \), including shadowing. We further introduce the function \( G_i(l, k) \).

\[
1G_i \text{ corresponds to all possible gain patterns of the beamforming antenna, which were obtained with a separate MatLab program from standard beamforming signal processing algorithms.}
\]
belong to the same cell sector, which leads to a modification of eq. (5):

\[
e_{kl} = \begin{cases} 
0 & \text{if } I_{kl} \in W_k \cap I_{ik} \in W_l \cap C(m_l) \neq C(m_k) \\
1 & \text{otherwise}
\end{cases}
\]

where \(C(m_l)\) denotes the cell sector serving mobile \(m_l\).

C. Characteristics of the Interference Graph

The degree of each vertex describes the amount of critical interference relations for each terminal. In areas with good reception conditions, the vertex degree is likely to be low, while it is expected to be high in areas with bad reception conditions. Furthermore, the vertex degree depends on the number of mobile terminals \(N\) and on the coordination diameter \(d_{ic}\). In [11], we showed that a coordination of only neighboring base stations achieves an almost as good performance with respect to the aggregate sector throughput as a global coordination. In the remainder of this paper, we will therefore only consider the case of \(d_{ic} = d_{BS}\) in order to limit the complexity of the resource assignment problem.

Figure 5 illustrates the vertex degree within the observation area (cmp. Fig. 1) for \(D_S = 15\) dB and \(N = 8\). The vertex degree is particularly low in the center areas of the cell sectors, which can be well covered by the beamforming antennas and which receive relatively small interference from neighboring sectors. In contrast, the degree is higher in the border areas of two cell sectors which are served by the same base station. They suffer from interference due to the relatively large width of the antenna beam and its significant side-lobes when the beam is directed to the side. The highest vertex degree can be observed in the border areas between cell sectors of neighboring base stations, where mobile terminals suffer from high inter-cell interference.

The probability density function (pdf) of the vertex degree is shown in Fig. 6 for different \(N\). The pdfs all show a similar behavior while demonstrating the expected higher vertex degree with an increasing number of mobile terminals \(N\). The pdfs show two obvious peaks, which can be correlated with the area representation in Fig. 5. The first peak corresponds to the innermost blue cell sector area, which can be well covered by the beamforming antennas. If the steering angle of the beamforming antennas is increased, the main beam spills over to the neighboring cell sector and the side-lobes become significant, leading to an increase of the interference and hence an increase in the vertex degree. This corresponds to the peak of the pdf and the red areas in Fig. 5. Note also that the vertex degree is always at least \(N - 1\).

IV. RESOURCE ASSIGNMENT BY GRAPH COLORING

A. The Graph Coloring Problem

During the resource assignment step, the constraints of the interference graph have to be taken into account. As all terminals which are connected in the interference graph must not be served by using the same set of resources, we have to find the minimum set of disjoint resources such that this requirement is met. This directly translates to the well-known graph coloring problem, in which the interference graph has to be colored such that no two connected vertices have the same color. The colors of the vertices correspond to particular resource sets, for example to disjoint sets of subchannels in the AMC-zone. Each set can then be used for the transmission towards a mobile terminal.

The number of colors \(M\) which are needed for the coloring determines the utilization of transmission resources in the system. If \(M\) colors are needed, \(M\) disjoint resource sets have to be formed. In each cell sector, only \(N\) of these resource sets will eventually be used, which implies a resource utilization of \(\rho = N/M\). In our problem, the minimum clique size of the
interference graph is equal to the number of mobile terminals $N$ per cell sector. Consequently, at least $N$ colors are needed. Most cases will require more colors, leading to a resource utilization below 100%.

The graph coloring problem is NP hard. In order to solve the coloring problem, we apply the heuristic Dsatur [12] and a tabu search technique [13]. While tabu search usually finds a smaller set of colors than the Dsatur algorithm, it is computationally much more expensive.

### B. Resource Utilization

As the minimum desired SIR $D_S$ is increased, the average vertex degree of the interference graph increases. This leads to an increase in the number of used colors and to a reduction of the resource utilization $\rho$. This is illustrated in Fig. 7, which plots $\rho$ over $D_S$. Shown are the results when applying Dsatur and tabu search, respectively. As expected, the resource utilization decreases if $D_S$ is increased, since more conflicts in the interference graph are produced and more colors are needed. Moreover, the computationally much more expensive tabu search produces a significantly better resource utilization.

Despite these expected results, the relationship between the number of mobile terminals $M$ and the resource utilization $\rho$ is not obvious. If we increase the number of mobile terminals $N$ in each cell sector, we also expect an increase in the number of required colors $M$. However, this does not allow any general conclusion for the ratio $\rho = N/M$, since the increase in $N$ may compensate for the increase of $M$. In fact, Fig. 7 shows that an increase in the number of mobile terminals from $N = 8$ to 12 or 16 leads to no significant changes in the resource utilization.

Figure 8 plots the pdf of the color indices, which are used for the graph coloring in average. For each individual coloring, the indices are sorted such that lower indices indicate more frequently used colors. The pdf reveals that the different colors are distributed unequally across the terminals. In particular, a small number of mobile terminals require extra colors in order solve the coloring problem. This decreases the resource utilization while at the same time it only supplies a small number of additional terminals. It is therefore worthwhile to consider the possibility of not serving all mobile terminals in each frame, but only a selection, possibly those terminals which are most favorable with respect to the graph coloring problem. In the following section, we will consider this issue among with certain other system imposed constraints.

### V. RESOURCE ASSIGNMENT IN PRACTICE

#### A. Impact of system constraints and QoS requirements

The evaluation in the previous section provided a basic analysis of the interference coordination problem and demonstrated the impact of the number of mobile terminals and the desired minimum SIR on the achievable resource utilization. It was assumed that all terminals are served in each radio frame. This will not be the case in a real system for several reasons. First, each transmission towards a terminal generates signaling overhead due to the transmission of the downlink map information in 802.16e and connection related signaling messages in the uplink direction. Second, the available resources are not arbitrarily divisible, and each transmission requires a certain minimum amount of granted resources due to the minimum ARQ block size, which cannot be further fragmented.

Another important system aspect is fairness. Compared to terminals in the cell center area, terminals located at the cell border areas are penalized in two ways. First, they suffer from worse SINR conditions due to a higher path-loss and a larger inter-cell interference, which lead to a lower throughput. Consequently, a higher amount of transmission resources is required to achieve a certain data rate. Second, they produce a larger number of conflicts in the interference graph (cmp. Fig. 5), which overproportionally increases the number of


B. Heuristic for combined Scheduling and Resource Assignment

Based on the previous considerations we present a heuristic which combines an arbitrary scheduling algorithm with the graph coloring problem. As discussed in the previous section, only a certain number $N_{srvd}$ of mobile terminals will be served in each frame. This allows to choose $N_{srvd}$ terminals out of the $N$ terminals in each cell sector according to a certain criteria. This may either be determined by a particular scheduling algorithm, but it may also be based on which terminals are best suitable to solve the graph coloring problem with a minimum number of colors. The latter case is related (but not identical) to the clique coloring problem for $N_{srvd} = 2$. A simple heuristic approach is to serve the terminals with the lowest vertex degree, since they produce the fewest conflicts.

In order to evaluate the impact of the scheduling algorithm, we will assign resources to $N_{sched}$ terminals based on the scheduler’s decision, and to the remaining $N_{srvd} - N_{sched}$ terminals based on their vertex degree. The flow-chart of the heuristic algorithm is shown in Fig. 9. The inner loop traverses all cell sectors in a random order and assigns resources to one terminal within every sector. The outer loop counts the number of rounds, where one round corresponds to the assignment of resources to one terminal in each cell sector. In the inner loop, the terminal to be served is selected based on the index of the current round. This is done such that in the first $N_{sched}$ rounds the terminal is selected according to the scheduler’s decision, and in all remaining rounds the terminal is selected based on its vertex degree. Whenever resources are assigned to a mobile terminal, these resources are blocked for all other mobile terminals, which are connected to it in the interference graph. Resources are assigned by dividing the AMC zone into $N_{srvd}$ equally sized disjoint sets of subchannels and assigning the first free and non-blocked set to the current terminal. The algorithm terminates once it has tried to assign resources to all mobile terminals.

C. Impact of Scheduling

As an example, a Random scheduling mechanism is used. The Random scheduler assigns the highest scheduling priority to each of the $N$ mobile terminals in a cell sector at least once within repeating periods of $N$ MAC-frames. Moreover,
the terminal with the highest priority in frame \( n \) becomes the mobile with the second highest priority in frame \( n+1 \), and so on. The considered Random scheduler therefore behaves very much like a Round Robin scheduler on short and long time scales.

Figure 10 plots the resource utilization \( \rho \) over \( D_S \) for different values of \( N_{sched} \). If we set \( N_{sched} \) to 0, the scheduler has no influence. Instead, terminals with favorable interference properties will be served preferably. In particular, the system will give preference to terminals with a low vertex degree in the interference graph, which increases the resource utilization at the cost of fairness. As \( N_{sched} \) is increased, more terminals are served based on the scheduler’s decision. This leads to a decrease in the resource utilization. However, this increases the fairness, which is illustrated in Fig. 11 and 12.

In Fig. 11, the fraction of radio frames \( f(\bar{x}) \) in which resources are assigned to a particular terminal depending on its position in the observation area is plotted, where \( \bar{x} \) denotes the geographic position of the terminal. A value of \( f(\bar{x}) = 0 \) means that a mobile terminal at the respective position is never served, while a value of \( f(\bar{x}) = 1 \) means that it is served in every radio frame. For \( N_{sched} = 0 \) we can see large areas at the cell border where mobile terminals are hardly ever served while in the center areas terminals are served in almost every frame. This implies that the coverage of the system is decreased. In contrast, for \( N_{sched} = 4 \), the assignment of resources to terminals is much less dependent on their position. The still higher serving probability in the cell center can be traced back to the circumstance that these terminals can more easily fill resource gaps after all terminals have been served based on the scheduler’s decision.

Further insight can be gained by looking at the distribution of \( f(\bar{x}) \). The complementary cumulative distribution function (ccdf) \( F(s) \) is calculated as

\[
F(s) = P(f(\bar{x}) > s).
\]

The ccdf \( F(s) \) is plotted in Figure 12. For \( N_{sched} = 0 \), the ccdf indicates a significant probability of about 6% for \( f(\bar{x}) = 0 \), i.e., for no coverage. On the other hand, in 10% of all locations, mobile terminals receive resources in 80% of all radio frames. As \( N_{sched} \) is increased, the coverage immediately increases to 100% of the cell sector area, and there are less areas where terminals are served excessively often. Ideally, this ccdf would be an inverted and shifted step function with the step at \( s = \min(1,N_{srvd}/N) \), which would indicate an equal assignment of resources to terminals independent of their position.

### D. Throughput Results

This section evaluates the achievable total sector throughput at the IP-level. Instead of Monte-Carlo simulations, we perform a full frame-level simulation with random direction mobility of the mobile terminals at a velocity of \( v = 30 \text{km/h} \) similar to [9]. The AMC zone was assumed to consist of 9 OFDM-symbols, corresponding to a total number of \( 48 \cdot 3 \) available subchannels. Adaptive Modulation and Coding was applied ranging from QPSK 1/2 to 64QAM 3/4, which results in a theoretical maximum raw data rate of about 6.2 Mbps within the AMC zone. Additionally, all relevant MAC-mechanisms, such as fragmentation, Automatic Repeat Request (ARQ) and Hybrid ARQ (HARQ) were modeled.

Figure 13 shows the total IP-level sector throughput over \( D_S \) for \( N_{srvd} = 4 \) and different values of \( N_{sched} \). As discussed in [9], the throughput shows a maximum for a particular \( D_S \), which is due to the tradeoff of the resource utilization and the achievable SINR. For \( N_{sched} = 4 \), the maximum total...
sector throughput is about 1453 kBit/s, while for $N_{sched} = 0$, it increases by almost 40% to about 2017 kBit/s, however at the expense of the before discussed coverage loss.

VI. CONCLUSION

In this paper, we solved the problem of interference coordination in cellular OFDMA networks by means of graph coloring. We showed that some colors during the graph coloring are only used by a few mobile devices, thus decreasing the resource utilization. Further, the number of necessary colors proved to be independent of the number of mobile terminals in the considered range, which implies an equal resource utilization independent of the number of mobile terminals. We also presented a heuristic which allows the combination of arbitrary scheduling algorithms with interference coordination. This heuristic allows to trade off scheduler requirements, such as Fairness and QoS constraints with the total sector throughput. It was seen that the total sector throughput can be increased by almost 40% at the cost of fairness and coverage.

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