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Balanced HDLC Procedures: A Performance Analysis

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Abstract—The prime objective of the present paper is to analyze the performance of HDLC Balanced Class of Procedures, i.e., to quantitatively study the interaction among a multiplicity of parameters which are procedure specific, characterize the properties of the transmission medium, and identify the operational characteristics and requirements. The approach taken is to consider two kinds of operation: a saturated case characterized by maximum throughput as the most suitable measure of performance, and a nonsaturated situation for which waiting and transfer times are the appropriate measures. The analysis is performed by means of both simulation and analytic techniques. Within the simulation model, the information transfer phase of the HDLC procedure was implemented in full detail. The key idea of the analytic approach is to use a so-called virtual transmission time, a quantity comprising both the real constant transmission time of a message and the duration of recovery actions in case of transmission errors. It allows the performance measures to be represented by explicit and easily-computable expressions. The results provide a fundamental insight into how the most relevant parameters interact and determine performance.

I. INTRODUCTION

The data-link control level in data and computer communication networks contains control functions such as addressing, frame numbering, error recovery, and flow control to reliably transfer data and control information across a physical communication link. Both national and international standardization bodies and computer manufacturers have defined data-link control procedures, e.g., ISO High-Level Data-Link Control Procedure (HDLC) [1]-[3], ANSI Advanced Data Communication Control Procedures (ADCCP) [4], Digital Equipment Digital Data Communications Message Protocol (DDCMP) [5], IBM Synchronous Data-Link Control Procedure (SDLC) [6].

The objective of the present investigation is to study the performance of HDLC when it is operated in the asynchronous balanced mode (ABM) over a point-to-point configuration. We are interested in: 1) identifying the essential parameters which determine protocol performance, and 2) analyzing their impact on performance under various conditions. In three recent papers [7]-[9] we provided detailed analyses of the balanced class of procedures through simulation techniques and analytic modeling, respectively; the present contribution combines the major results obtained there and represents a comprehensive performance analysis of the HDLC balanced class of procedures.

The protocol was implemented in full detail within the framework of a simulation study [7], the results of which provided a basic insight into the interaction among procedure-specific parameters, parameters characterizing the transmission medium, and the required operational parameters. The analytic model—structured such that it reflects all relevant parameters and mechanisms—uses the new concept of a virtual transmission time, a quantity comprising both the real transmission time of an information frame and the duration of recovery actions in case of transmission errors.

Related performance investigations of full-duplex data links are given in [10] for the throughput behavior of SDLC and the Unbalanced Classes of HDLC, and in [11], [12] for
throughput and transfer time of an idealized HDLC-like protocol. Furthermore, the impact on the transfer time of a finite window size (i.e., finite modulus) is analyzed in [13] for error-free links with negligible processing and propagation delays. In [14], distributions of queue lengths and waiting times are derived for a slotted statistical multiplexer with an ARQ retransmission scheme. The major difference between our investigation and known analyses of link-control procedures is that our model is intended to reflect the mechanisms of an actual link-control procedure (HDLC ABM) as closely as possible.

In the next section, we briefly review the main features of HDLC balanced class and explain the data-link model underlying the performance evaluation of the procedure. In Sections III and IV, we describe the basic ideas of the concept employed in the analysis of maximum throughput and mean transfer time, respectively. Section V contains a discussion of results.

II. HDLC BALANCED CLASS OF PROCEDURES—DATA-LINK MODEL

A. Background Information on the Procedure

Balanced operation is intended for situations which require equal control capability at both ends of a point-to-point link. The stations at both sides of the link are so-called combined stations which means that each station can send and receive both commands and responses. (The classes of procedures for unbalanced operation, conversely, have two types of stations: primary and secondary.)

All transmissions are in frames and each frame conforms to one of the formats shown in Fig. 1(a) and (b). Frames transporting information, I-frames, are sequentially numbered with their send sequence numbers N(S), which cycle through the set \( \{0, 1, 2, \ldots, M - 1\} \) where \( M \) is the modulus of sequence numbers. The modulus equals eight for the unextended and 128 for the extended control-field format, respectively. Frames containing supervisory control sequences only, S-frames, have the same structure but no information field. The acknowledgment function is realized by the receive sequence number \( N(R) \) contained in all I- and S-frames. The station transmitting a certain value of \( N(R) \) indicates that it has correctly received all I-frames numbered up to \( N(R) - 1 \).

For the particular purpose of this performance study, we assume: 1) that setting up the link has been appropriately handled, and 2) that the link established is available and operational for the time interval required. The following commands and responses from the basic repertoire are used: Information (I) and Receive Ready (RR). In addition, we use the command/response Reject (REJ) being offered under optional functions for improved performance.

To illustrate how these commands/responses are used in the model to be discussed in Section II-B, we subsequently outline various characteristic patterns of operations.

1) Error-Free Operation: The simplest case occurs when both stations have I-frames ready for transmission all the time. Then I-frames can be used to acknowledge reception. In the case where a station does not have any more I-frames to be sent, it can acknowledge incoming frames by sending an RR-frame.

2) Operation with Errors: I-frames with a frame-check sequence (FCS) error are discarded and have to be retransmitted. The error will manifest itself later in the form of a sequence error, or it will be detected by the expiration of a timeout. We can differentiate among three mechanisms to recover errors:

a) Checkpoint Retransmission (P/F-Bit Recovery): HDLC provides a checkpointing function in ABM by using Poll and Final (P/F) bits. As the P- and F-bits are always exchanged as a pair (for every P there is one F, and the P cannot be issued until the previous P has been matched with an F), the \( N(R) \) count contained in a frame with the F-bit set to 1 can be used to detect I-frame sequence errors.

b) REJ Recovery: The REJ command/response is used to initiate an earlier retransmission following the detection of a sequence error than is possible by checkpointing.

c) Time-Out Recovery: A single I-frame or the last I-frame in a sequence of I-frames cannot be recovered by REJ. Also, a frame with the P-bit set to one may be lost. Furthermore, it is not allowed to repeat the REJ recovery if a retransmitted I-frame is again disturbed. Therefore, each combined station uses a timer to recover from such situations. Since it can happen that an I-frame has been correctly received, but the acknowledgment has either not been sent or lost, an RR command with the P-bit set to one is issued upon expiration of the timeout prior to retransmission to avoid a duplication of the I-frame already sent.

B. The Model

A schematic representation of the model underlying the performance study is shown in Fig. 2. It consists of two data stations connected by a full-duplex circuit. The link is controlled by HDLC balanced class of procedures including the optional function REJ. Messages to be transmitted from station \( A \) to \( B \), or vice versa, are stored in the send buffer of the sending station where they have to wait for transmission. Messages are transmitted according to first-come/first-served, one message per I-frame. Throughout this paper we assume that the size of the message buffers is unlimited and that the messages are of constant length \( l \). The transmission channels are characterized by their transmission rate \( u \), their bit-error probability \( p_{bit} \) (independent bit errors), and their (one-way) propagation delay \( t_{prop} \). Furthermore, we assume that for the processing of a received frame a constant time \( t_{proc} \) is required. For the analysis we combine propagation...
and processing delay in a constant but arbitrarily selectable delay

\[ t_p = t_{\text{proc}} + t_{\text{prop}}. \]  

Furthermore, the following abbreviations concerning parameters of the model will be used: \( p_B \): block-error probability of I-frames, i.e., probability that an I-frame contains at least one bit-error; \( t_f \): transmission time of an I-frame \( t_f = (l + 48) \text{bit/v in case of } M = 8 \); \( t_s \): transmission time of an S-frame \( t_s = 48 \text{bit/v in case of } M = 8 \).

III. THROUGHPUT ANALYSIS

In this section, we derive analytic expressions for the maximum information throughput of a link. Apart from its theoretical significance, maximum throughput as a function of the relevant system parameters is particularly interesting for batch applications. It also represents an upper bound on the traffic which the link can carry without the queues becoming unstable.

The following considerations pertain to one direction of transmission, namely, from station A to B. Maximum throughput in this direction of transmission is achieved if station A has information to be sent at any time whereas channel B to A is idle. This is due to the fact that both error-recovery and acknowledgment handling suffer minimum delay if channel B to A is idle. The acknowledgment time \( t_{\text{ack}} \) between the end of the successful transmission of an I-frame and the receipt of the acknowledgment is constant in this case and given by (cf. Fig. 3)

\[ t_{\text{ack}} = 2t_p + t_s. \]  

For clarity’s sake we describe the throughput analysis under the above assumption. Note, however, that the throughput analysis can be analogously performed for the case where this condition is not fulfilled. In [9] it is demonstrated that the impact of a fully loaded channel B to A can be taken into account with sufficient accuracy by appropriately adjusting the mean value of the acknowledgment time. Throughout the analysis it is assumed that the probability of an S-frame being in error can be neglected. (The S-frame length is 48 bit for the unextended and 56 bit for the extended control-field-format.)

A. The Concept of the Virtual Transmission Time

We address the problem by using the notion of a virtual transmission time defined in the following way: The virtual transmission time of an I-frame with \( N(S) = i \) begins with the start of its transmission provided the I-frame with \( N(S) = i - 1 \) is received in sequence and without transmission error. It terminates at the end of its transmission at the sending station, provided this transmission of the frame is successful. If the I-frame with \( N(S) = i \) cannot be transmitted due to \( M - 1 \) unacknowledged I-frames, its virtual transmission time is prolonged by the time it has to wait until “the window opens again” (cf. Fig. 3 and Section III-B2).

This definition enables us to replace the complicated sequence of I- and S-frames in case of errors by an equivalent but much simpler sequence of virtual transmission times. The expectation \( t_v \) of the virtual transmission time corresponds to the mean time required to successfully transmit one I-frame from station A to station B. Therefore, the maximum information throughput \( T \) is given by

\[ T = \frac{1}{t_v}. \]  

To determine the mean virtual transmission time in the saturated case, we distinguish between the following two cases pertaining to the relation among acknowledgment time \( t_{\text{ack}} \), transmission time of an I-frame \( t_f \), and modulus \( M \) of the sequence numbers: In Case 1) the acknowledgment time \( t_{\text{ack}} \) is greater than the time to transmit \( M - 2 \) I-frames (cf. Fig. 3), whereas in Case 2) the time \( t_{\text{ack}} \) is not greater than \( (M - 2) \cdot t_f \).

B. Case 1): \( t_{\text{ack}} > (M - 2)t_f \)

1) Principle of Analysis: To cope with the interaction among the finite modulus \( M \) and transmission errors, we consider a random variable \( W \) denoted as window width. The window width \( W \) is defined at the beginning of each virtual transmission time and corresponds to the number of I-frames which station A will subsequently continuously transmit until it must stop because of \( M - 1 \) acknowledged I-frames.

We denote by \( t_v(w) \) the mean virtual transmission time of I-frames for which the window width \( W \) equals \( w \) at the beginning of their virtual transmission times, and by \( \phi(w) \) the probability that \( W \) equals \( w \). Then the mean virtual transmission time can be written as

\[ t_v = \sum_{w=0}^{M-1} \phi(w)t_v(w). \]  

In Sections III-B2 and III-B3 we derive analytic expressions
The Window-Width Process.} Fig. 3 illustrates how transmission errors and the modulus interact and affect the virtual transmission time of the I-frames. In particular, it shows the values of the window width \( W \) defined at the beginning of the virtual transmission times. For the sake of a simplified representation, the modulus \( M \) is equal to four in Fig. 3. We can see that due to the specific definition of the virtual transmission time, channel \( A \) to \( B \) is virtually occupied without any time gap in the saturated case, i.e., the virtual transmission times \( T_\alpha(2) \), \( T_\alpha(1) \), \( T_\alpha(0) \), \( T_\alpha(2) \) are contiguous. The window width \( W \) as defined above can only have the values \( 0, 1, 2, \ldots, M-2 \), but not \( M-1 \), because, at the beginning of a virtual transmission time, the preceding I-frame cannot yet have been acknowledged.

Under our assumptions, the window-width process forms a Markov chain which is illustrated in Fig. 4 and described as follows: the window width \( W \) is reduced by one every time an I-frame is successfully transmitted (probability \( 1 - p_B \)) until the minimum achievable value of \( W = 0 \) is reached. Note, we consider the case \( t_{\text{ack}} > (M - 2)t_f \) and I-frames which see a window width \( W = 0 \) are delayed for a certain time \( t_d \) until they can be transmitted due to an acknowledgment received. As can be seen from Fig. 3, the delay \( t_d \) is given by

\[
t_d = t_{\text{ack}} - (M - 2)t_f.
\]

Following a disturbed I-frame, e.g., I-frame with \( N(S) = 3 \) in Fig. 3, the window width returns to \( M - 2 \) by definition the next observation epoch of the window-width process is located just after the recovery action is finished. The transition from state \( W = 0 \) to \( W = M - 2 \) in case of no transmission error (probability \( 1 - p_B \)) can be explained as follows. Consider an I-frame for which the window width at the beginning of its virtual transmission time equals zero. Since, by the definition of the virtual transmission time, all predecessors of this frame have been or will be received error-free, it is the first in a series of exactly \( M - 1 \) continuously-transmitted I-frames. Therefore, following this frame, station A will continuously transmit exactly \( M - 2 \) I-frames before it has \( M - 1 \) unacknowledged I-frames outstanding. In other words, at the beginning of the next virtual transmission time, the window width as defined above, has the value \( M - 2 \).

From the state-transition diagram of the window-width process shown in Fig. 4, the probabilities \( \phi(w) \) can be easily determined:

\[
\phi(w) = \frac{p_B(1 - p_B)^{M-2-w}}{1 - (1 - p_B)^{M-1}} \quad \text{for } w \in \{0, 1, \ldots, M-2\}.
\]

3) Conditional Expectation Values of the Virtual Transmission Time. With probability \( 1 - p_B \) the I-frame considered is not disturbed. Since an I-frame which sees a window width \( w = 0 \) is delayed for a time \( t_d \) until the window opens again, the virtual transmission time \( T_v(w) \) is given by (see Fig. 3)

\[
T_v(w) = T_0(w) = \begin{cases} 
  t_f + t_d & w = 0 \\
  t_f & w \in \{1, 2, \ldots, M-2\}.
\end{cases}
\]

With probability \( p_B^N(1 - p_B) \), exactly \( n \) transmissions of the I-frame considered are disturbed before it is correctly received. As described in Section II-A, if the retransmission of an I-frame is disturbed, then the error situation is resolved by timeout recovery. Therefore, in this case \( T_v(w) \) is equal to (cf. Fig. 3)

\[
T_v(w) = T_0(w) + T_1(w) + (n-1)t_2.
\]

The components \( T_1(w) \) and \( t_2 \) in (8) are, in general, defined as follows. The time interval \( T_1(w) \) begins immediately after the transmission of the considered and disturbed I-frame which at the beginning of its virtual transmission time saw a window width \( w \). It ends immediately after the first retransmission of this frame. The time interval \( t_2 \) begins immediately after a retransmission of the I-frame considered. It ends immediately after the next retransmission of this frame.

From (7) and (8), the conditional expectation of the virtual transmission time is simply derived:

\[
t_v(w) = E[T_v(w)] = E[T_0(w)] + p_B E[T_1(w)] + \frac{p_B^2}{1 - p_B} t_2.
\]

For the derivation of \( E[T_1(w)] \), we have to distinguish carefully between the situations where recovery is performed via REJ and timeout. The choice between these two possibilities depends on the actual value of the window width and whether none, or one, or even more of the I-frames following the one considered are also disturbed. As shown in [15], the following expressions for the expectation of the time component \( T_1(w) \) can be derived:

\[
E[T_1(0)] = t_f \sum_{x=0}^{M-3} (x + 1)(1 - p_B)p_B^x + p_B^{M-2}(t_{\text{out}} + t_2) + t_{\text{ack}} + t_f.
\]
The value of the time component \( t_2 \) is independent of the window width \( w \) and simply given by
\[
t_2 = t_{\text{out}} + t_z + t_{\text{ack}} + t_f.
\]
(12)

C. Case 2: \( t_{\text{ack}} \leq (M-2)/t_f \)

The main difference from Case 1) is that here the virtual transmission time no longer depends on the actual value \( w \) of the window width. Therefore, in analogy to (9), the mean virtual transmission time is given by
\[
t_v = E[T_v] = t_f + p_B E[T_1] + \frac{p_B^2}{1-p_B} t_2.
\]
(13)

To determine the expectation of the time component \( T_1 \) and the interval \( t_2 \), we must visualize that there is an upper-bound \( c \) for the number of I-frames which-following a disturbed I-frame—can be transmitted before recovery is started. The value of \( c \) is either given by \( M - 2 \), because of the modulus rule, or by the number of I-frames which can be transmitted until the timeout expires. More precisely, \( c \) is equal to the least of these numbers:
\[
c = \inf \left\{ M - 2, \left\lceil \frac{t_{\text{out}}}{t_f} \right\rceil + 1 \right\},
\]
(14)

where \( \lceil \cdot \rceil \) is defined as the greatest integer not exceeding \( \gamma \).

1) Time Component \( T_1 \): As in Case 1), we have to distinguish between recovery via REJ and recovery via timeout. Recovery via REJ is performed if, following the one considered, not all \( c \) I-frames are disturbed. Equation (15) which holds with probability \( (1 - p_B) \cdot P_d \cdot \Xi(0, 1, 2, ..., c - 1) \), represents the two situations to be distinguished in case of REJ recovery:
\[
T_1 = \tau(x) = \begin{cases} 
(x + 1)t_f + t_{\text{ack}} + t_f & \text{if } (x + 1)t_f + t_{\text{ack}} > ct_f \\
(x + 1)t_f + \left\lceil \frac{t_{\text{ack}}}{t_f} \right\rceil + 1 \ t_f + t_f & \text{if } (x + 1)t_f + t_{\text{ack}} \leq ct_f.
\end{cases}
\]
(15)

The following comments, supported by Fig. 5, provide a physical explanation:

a) Upon receipt of the REJ-frame, channel A to B is idle either because of timeout expiration or because the maximum number of \( M - 1 \) simultaneously unacknowledged outstanding I-frames, has been reached. This case corresponds to the upper line in (15).

b) Upon receipt of the REJ-frame an I-frame is currently being transmitted from A to B. Since we adopted the strategy not to abort the transmission of an I-frame if REJ is received, the time \( T_1 \) is an integral multiple of the I-frame transmission time \( t_f \) in this case and given by the lower line of (15).

Time-out recovery is performed if all \( c \) I-frames following the one considered are disturbed. Equation (16) which holds with probability \( p_B^c \) represents the two situations to be distinguished in this case:
\[
T_1 = \tau(c) = \begin{cases} 
ct_f + t_z + t_{\text{ack}} + t_f & \text{if } t_{\text{out}} \leq ct_f \\
t_{\text{out}} + t_z + t_{\text{ack}} + t_f & \text{if } t_{\text{out}} > ct_f.
\end{cases}
\]
(16)

The following comments interpret (16):

a) If \( t_{\text{out}} \leq ct_f \) then the timeout expires during the transmission of the \( c \)th I-frame following the one considered. In this case, checkpointing with an RR command with the P-bit set to 1 is performed immediately after transmission of this I-frame. This situation is reflected in the upper line of (16).

b) If \( t_{\text{out}} > ct_f \) then checkpointing can be performed immediately after the expiration of the timeout because channel A to B is certainly idle. This case corresponds to the lower line in (16).

From (15) and (16) the expectation of \( T_1 \) in Case 2) can be simply computed:
\[
E[T_1] = \sum_{x=0}^{c-1} (1 - p_B) P_d \Xi(x) + p_B^c \Xi(c).
\]
(17)

2) Time Component \( t_2 \): Since after a disturbed retransmission timeout recovery is performed, we obtain the same result for the time \( t_2 \) as for \( T_1 \) in case of timeout recovery [see (16)]:
\[
t_2 = \begin{cases} 
ct_f + t_z + t_{\text{ack}} + t_f & \text{if } t_{\text{out}} \leq ct_f \\
t_{\text{out}} + t_z + t_{\text{ack}} + t_f & \text{if } t_{\text{out}} > ct_f.
\end{cases}
\]
(18)
IV. TRANSFER-TIME ANALYSIS

In contrast to the considerations in Section III, we assume now that the channels are only loaded corresponding to a fraction of their full capacity and that the traffic on both channels varies statistically. This situation is typical of interactive traffic where the transfer time of the messages transmitted represents the essential measure of performance. The transfer time is defined here as the time from the arrival of a message at one station until its successful receipt at the other station.

Besides traffic conditions on the link, the following main parameters may influence the transfer time of messages and are included in the performance model: channel characteristics (transmission rate, propagation delay, bit-error probability) and processing delays.

The modulus, generally speaking, has a significant impact on performance only if the round-trip delay-message and acknowledgment—is such that sufficient time is available within this time interval to transmit $M - 2$ I-frames. For interactive applications over terrestrial links such a situation is very unlikely to occur. Therefore, the transfer-time analysis subsequently described covers the most important range of applications in case of terrestrial links, although the impact of a finite modulus is not taken into consideration. In addition, it holds for satellite links provided the modulus is sufficiently high. The simulation model, of course, allows the transfer time to be studied also in those cases where the impact of the modulus cannot be disregarded.

A. Principle of Analysis

We assume that messages to be transmitted to station $B$ or $A$ arrive at station $A/B$ according to a Poisson process with rate $\lambda_B > 0, \lambda_A > 0$, such that the total channel utilization is less than one.

Again, the considerations pertain to one direction of transmission, namely, from station $A$ to station $B$. Due to the balanced class of procedures, the results for the other direction can be obtained in the same way.

The transfer-time analysis is also based on the notion of the virtual transmission time, as defined in Section III. This approach allows again the complicated sequence of I- and S-frames to be replaced in case of transmission and sequence errors by the much simpler sequence of the virtual transmission times, because we can conceive that the I-frames transmitted occupy the channel for the duration of their virtual transmission times. Therefore, the mean transfer time of messages can be determined by using the Pollaczek-Khintchine formula [16]:

$$ t_f = \frac{\lambda_A E[T_e^2]}{2(1 - \lambda_A E[T_e])} + E[T_e] + t_p. \tag{19} $$

The main problem is to determine the first two moments of the virtual transmission time $T_v$. This is described in the following section.

B. Virtual Transmission Time

1) Decomposition of the Virtual Transmission Time: As shown in Fig. 6, we decompose the virtual transmission time of a disturbed I-frame into appropriate components. In particular, we define two random time intervals $T_e$ and $T_r$ as follows.

- The time interval $T_e$ begins at the end of transmission of the considered (disturbed) I-frame at station $A$. It ends at the receipt of the first error-free I-frame at station $B$ following the one considered, or the receipt of an RR frame with the $P$-bit set to one, following the expiration of the timeout.

- The time interval $T_r$ begins at the receipt of the first error-free I-frame following the one considered or the receipt of the RR frame with the $P$-bit set to one, following the expiration of the timeout. It ends when the retransmission of the I-frame considered begins.

If the I-frame considered is received without transmission and sequence error, its virtual transmission time is equal to $t_r$. If $N$ transmissions of the I-frame considered are disturbed ($N \geq 1$), its virtual transmission time is composed as follows (cf. Fig. 6):

$$ T_v = 2t_r + T_e + T_r + (N - 1)(t_l + t_{out} + t_s + t_p + t_r). \tag{20} $$

Since the number of transmissions $N$ is geometrically distributed with parameter $p_B$, the first two moments of $T_v$ are given by

$$ E[T_v] = \frac{1}{1 - p_B} t_r + p_B E[T_e] + \frac{p_B}{1 - p_B} E[T_r] 
+ \frac{p_B}{1 - p_B} (t_{out} + t_s + t_p). \tag{21} $$

$$ E[T_v^2] = (1 + 3p_B)t_r^2 + p_B E[T_e^2] + \frac{p_B^2 (1 + p_B)}{(1 - p_B)^2} \theta^2 
+ \frac{p_B}{1 - p_B} E[T_r^2] + 4t_r \left( \frac{p_B}{1 - p_B} \right) \left( \frac{p_B^2}{1 - p_B} \right) \theta 
+ \frac{p_B}{1 - p_B} E[T_r] \left( \frac{p_B}{1 - p_B} \right) \left( \frac{p_B^2}{1 - p_B} \right) \theta \left( E[T_r] \right) 
+ 2E[T_e] \left( \frac{p_B}{1 - p_B} \right) \left( \frac{p_B^2}{1 - p_B} \right) \theta \left( E[T_e] \right) 
+ \frac{2p_B^2}{(1 - p_B)^2} E[T_r]^2. \tag{22} $$

with

$$ \theta = t_{out} + t_l + t_s + t_p. $$
For (22), it is assumed that $T_e$ and $T_r$ are stochastically independent. This assumption is not valid in a strict sense, but it represents an acceptable working basis, as can be seen from a comparison with simulation results.

In the following two sections, it is shown how the first two moments of $T_e$ and $T_r$ are required to evaluate $E[T_e]$ and $E[T_r^2]$ in (21) and (22) can be determined.

2) Time Component $T_e$: The definition of the time component $T_e$ was given in Section IV-B1. To determine its first two moments, we proceed as follows. Let $\xi_e$ be the time between end of transmission of the I-frame considered with $N(S) = i$ and the end of transmission of the I-frame with $N(S) = i + x \pmod{M}$. We define a new random variable $T_{e,x}$ by

$$T_{e,x} = \begin{cases} \xi_e + t_p, & \xi_e \leq t_{out} + t_f \\ t_{out} + t_S + t_p & \text{otherwise.} \end{cases}$$

(23)

As can be seen from Fig. 7, $T_{e,x}$ is equal to $T_e$, provided the I-frame with $N(S) = i + x \pmod{M}$ is the first I-frame without transmission error following the I-frame considered with $N(S) = i$. Furthermore, Fig. 7 illustrates the two cases which must be distinguished: if $\xi_e < t_{out} + t_f$ then the first error-free I-frame is transmitted before the timeout expires (REJ recovery), otherwise the timeout expiration initiates error recovery.

Using the random variables $T_{e,x}$ defined in such a way, the distribution and the first two moments of $T_e$ can be written as

$$P(T_e \leq t) = \sum_{x=1}^{k+1} (1 - p_B) p_B^{x-1} P(T_{e,x} \leq t),$$

(24)

$$E[T_e] = \sum_{x=1}^{k+1} (1 - p_B) p_B^{x-1} E[T_{e,x}] + p_B^{k+1} (t_{out} + t_S + t_p),$$

(25)

$$E[T_e^2] = \sum_{x=1}^{k+1} (1 - p_B) p_B^{x-1} E[T_{e,x}^2] + p_B^{k+1} (t_{out} + t_S + t_p)^2,$$

(26)

The random variable $\xi_e$ defined above can be conceived as the time until the $x$th departure of a customer from an $M/D/1$ queue measured from a departure epoch. The distribution $F_x(t) = P(\xi_e < t)$ has been derived by Pack [17]. Using this result, the following expressions for the quantities $E[T_{e,x}]$ and $E[T_{e,x}^2]$, which are needed to evaluate the first two moments of $T_{e,x}$, can be derived (for details see [15]):

$$E[T_{e,x}] = t_{out} + t_S + (t_f - t_S) F_x(t_{out} + t_f) + F_x^*(x(t_f)) - F_x^*(t_{out} + t_f + t_p),$$

(27)

$$E[T_{e,x}^2] = (t_{out} + t_S + t_p)^2 + 2 t_p (t_f - t_S) F_x(t_{out} + t_f)
+ \left[(t_{out} + t_f)^2 - (t_{out} + t_S)^2\right] F_x(t_{out} + t_f)
+ 2(t_{out} + t_f + t_p) F_x^*(t_{out} + t_f)
+ 2 F_x^{**}(t_{out} + t_f)
+ 2(t_f + t_p) F_x^*(x(t_f)) - 2 F_x^{**}(x(t_f)),$$

(28)

with

$$F_x^*(t) = \int_{t_f}^{\infty} F_x(t) dt, F_x^{**}(t) = \int_{t_f}^{\infty} F_x^*(t) dt.$$

In [15] it is also shown that simple recursive relations hold for the integral expressions $F_x^*$ and $F_x^{**}$ in (27) and (28). Therefore, $E[T_e]$ and $E[T_r^2]$ can be straightforwardly computed by substituting (27) and (28) into (25) and (26).

3) Time Component $T_r$: The general definition of the time $T_r$ is given in Section IV-B1. According to Fig. 6, we decompose $T_r$ into four components:

$$T_r = T_{res,B} + t_S + T_{res,A} + t_p.$$  

(29)

Assuming independence of $T_{res,A}$ and $T_{res,B}$ the first two
momenos of $T_r$ are given by

$$E[T_r] = E[T_{res,A} + t_s + E[T_{res,B}] + t_p],$$

$$E[T_r^2] = E[T_{res,A}^2 + t_s^2 + E[T_{res,B}^2] + t_p^2 + 2E[T_{res,A}](t_s + E[T_{res,B}] + t_p) + 2E[T_{res,B}](t_s + t_p) + 2t_s t_p].$$

To estimate the first two moments of $T_{res,A}$ and $T_{res,B}$, we assume that the arrival times at station $B$ of the first correct I-frame following the one considered and the arrival times of the RR frames with the P-bit set to one following a timeout are purely random, and that the same holds for the arrival times at station $A$ of the REJ frames or the RR frames with the F-bit set to one. Then $T_{res,A}$ and $T_{res,B}$ can be considered as residual lifetimes of a renewal process with constant interevent time $t_f$, if the channel is occupied. This consideration leads to

$$E[T_{res,A}] = \lambda_A t_f^2/2, \quad E[T_{res,B}^2] = \lambda_A t_f^3/3,$$

$$E[T_{res,B}] = \lambda_B t_f^2/2, \quad E[T_{res,B}^2] = \lambda_B t_f^3/3.$$ 

V. NUMERICAL RESULTS

In consonance with the considerations in Sections III and IV, there are basically two categories of results: maximum throughput of information bits and average transfer time.

A. Throughput Results

Figs. 8 and 9 represent typical calculation and simulation results for the throughput characteristic of an HDLC link as a function of the essential parameters: message length $l$, processing plus propagation delay $t_p$, modulus $M$, transmission rate $v$, and bit-error probability $p_B$.

Fig. 8 for a terrestrial link shows the throughput efficiency, maximum throughput of information bits per second $T$ relative to the transmission rate $v$ as a function of the message length $l$, i.e., the information field length of the I-frames. Two different situations are studied: 1) only channel A to B is fully loaded whereas the reverse channel does not carry any information bits, and 2) both channels are assumed to be fully loaded. (Note that the figure only shows the throughput from station A to B.)

The curves for $v = 4.8$ kbits/s show the typical throughput behavior of link control procedures employing an error detection and retransmission strategy [18]: for short message lengths the throughput is low due to the relatively large overhead of flag, address, control, and frame checking sequence bits (48 bits in the case of $M = 8$). For longer message lengths, the relative overhead decreases but the block-error probability $p_B$ increases according to

$$P_B = 1 - (1 - p_0)^{48 l}.$$ 

Therefore, the throughput curves show a maximum.

An additional characteristic—particularly pronounced for $v = 48$ kbits/s—is the impact of the modulus causing a drastic throughput degradation for short message lengths. The reason for this behavior is the aforementioned HDLC rule that a station must stop sending further I-frames if it has seven unacknowledged I-frames outstanding.

A further remarkable feature of the throughput curves in case of an idle channel B to A is that they show distinct discontinuities at those message lengths for which the acknowledgment time $t_{ack}$ is an integral multiple of the I-frame transmission time $t_f$ and $t_{ack} \leq (M - 2)t_f$. The physical explanation of this effect is as follows: if, in case of a disturbed I-frame, the REJ has been received and processed just before the end of an I-frame transmission, then the additional delay due to the residual transmission time of this frame is short. This takes place if $t_{ack}$ is slightly shorter than an integral multiple of $t_f$. If, however, $t_{ack}$ is slightly longer than an integral multiple of $t_f$, the virtual transmission time of a disturbed I-frame is prolonged by the residual transmission time of an I-frame which is almost equal to $t_f$ in this case. However, this effect strongly depends on the following assumptions: 1) constant I-frame length, 2) idle channel B to A, and 3) no abort of transmission upon receipt of REJ.
sequently, in case of a fully loaded channel \( B \) to \( A \) such discontinuities do not exist. The analytic results for the latter case have been gained by using the analysis in [9] which is-as described in Section III-completely analogous to the approach described in this paper. Due to the prolongation of acknowledgment and recovery times the throughput efficiency of channel \( A \) to \( B \) is slightly reduced if channel \( B \) to \( A \) is fully loaded as compared to the case of an idle channel \( B \) to \( A \).

The explanation of the throughput behavior of the satellite link in Fig. 9 is virtually the same. Two bit-error probabilities are considered, \( 10^{-7} \) and \( 10^{-6} \), where the higher of these values may be caused by a poor-quality terrestrial extension of the satellite channels. Of course, the impact of the modulus is more distinct due to the long propagation delay of the satellite channels. Increasing the modulus from 8 to 128 yields a substantial improvement of the throughput as shown for the case of \( P_{bit} = 10^{-5} \). However, as can be seen from the reference curve for the terrestrial link with \( M = 8 \) and \( P_{bit} = 10^{-5} \) (dashed line) we do not obtain the maximum throughput of the terrestrial link by this means. The reason for this effect is that the error recovery takes longer for longer processing and propagation delays. Formally expressed, this dependence is mainly due to the term \( T_{1} E[T_{1}] \) in (17) where the expectation of \( T_{1} \) includes twice the processing and propagation delay \( t_{p} \) [cf. (15), (16), and (5)].

**B. Transfer-Time Results**

Here, we consider the case where the transmission channels are only loaded to a fraction of their full capacity.

Fig. 10 represents the mean transfer time \( t_{p} \) for messages versus the message length \( l \) for three transmission rates. It is assumed that the useful load of both channels is constant and equal to 0.6. The useful channel load represents that portion of the total channel utilization which is caused by the successful transmission of information bits

\[
Y_{u} = \lambda_{A} l/v. \tag{35}
\]

The curves show distinct minima which can be globally explained by the large relative overhead per I-frame for short messages, and increase of the virtual transmission time for long messages. The latter effect is caused because: 1) the real transmission time increases, 2) the block-error probability increases, and 3) the error recovery takes longer.

Whereas in the simulation model the HDLC procedure was fully implemented including the finite modulus, the impact of the modulus rule is not taken into account in the transfer-time analysis, as described in Section IV. Comparing analytic and simulation results we can observe that the modulus \( M = 8 \) has no impact on the transfer time for the transmission rates of 4.8 and 9.6 kbit/s over the whole range of message lengths which are of practical interest. In case of \( v = 48 \) kbit/s and \( M = 8 \) the modulus rule of HDLC causes a steep increase of the mean transfer time for message lengths of less than 600 bit (dashed line). Increasing the modulus to \( M = 128 \) yields a substantial improvement of the transfer time in this range because the modulus 128 no longer affects the transfer time, as can be seen from the accordance of simulation and analytic results.

Taking into account possible traffic overload situations as well as uncertainties in identifying the link parameters in practice (bit-error probabilities, propagation, and processing delays) the following conclusion can be drawn from Fig. 10: it is advisable not to operate an HDLC-controlled link in a parameter range where the modulus may have a significant impact, at least if the transfer time of the messages is of interest.

Fig. 11 represents the mean transfer time as a function of the useful channel load. It should be noted that the increase of the mean transfer time due to transmission errors is significant even for a rather small bit-error probability of \( 2 \times 10^{-6} \) corresponding to a block-error probability 0.01. This is again caused by the multiplicative interaction of block-error probability and processing plus propagation delay. Here, the performance degradation is even more pronounced than in the saturated case because—in terms of our analytic approach—a higher value of \( t_{p} \) not only increases the mean of the virtual transmission time but also its coefficient of variation. This in turn leads to longer waiting and transfer times [see (19)].

**VI. CONCLUSIONS**

The prime objective of our investigation was to model all relevant mechanisms of an actual data link control procedure, namely, the asynchronous balanced mode of HDLC rather than to analyze an idealized protocol. The major conclusions are:

1) It is possible to evaluate the performance of the balanced class of procedures of HDLC for a wide range of parameters by using a rather simple queuing model. The key to the analysis is the concept of virtual transmission time.

2) Comparison of the analytic results with results obtained from the simulation of a data link, where the protocol was fully implemented, indicate that through the new approach the major effects determining protocol performance can be adequately modeled.
3) The notion of the virtual transmission time provides a sound explanation of the performance properties of HDLC links, such as the multiplicative effects of growing processing and propagation delays, and growing block-error probabilities of information frames.

4) The HDLC balanced class of procedures allows efficient utilization of the transmission links and low waiting and transfer times, provided the relevant parameters are adjusted to meet the needs of the specific application.

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REFERENCES


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