CALL CONGESTION IN LINK SYSTEMS 
WITH INTERNAL AND EXTERNAL TRAFFIC

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ABSTRACT
In modern telephone exchanges the subscriber selection network is generally a multistage link system with both-way traffic, i.e. not only calls are offered to the link system which occupy one path through the link system (external traffic) but also such calls are offered which occupy two paths through the link system (internal traffic). An internal traffic call originates from a certain inlet of the link system and is connected to another inlet (subscriber) of this link system, i.e. the call needs two paths through the link system, an outgoing and an incoming path. In private automatic branch exchanges (PABX) link systems with internal and external traffic can also be found.

For the approximate calculation of the call congestion of such link systems a method is presented for PCT 1 (pure chance traffic No. 1, i.e. Poisson input having constant call intensity and negative exponential holding time distribution). This calculation is based on the method of combined inlet and route blocking (CIMB) [4, 5, 7]. Furthermore, this method makes use of the distribution function of the probabilities of state which was derived in [1, 2] for full available groups with internal and external traffic.

In Chapter 1 the principle of the calculation method is explained. In Chapter 2 the internal and external traffic is characterized. In Chapter 3 the different operation modes of link systems with internal and external traffic are discussed. The various formulae of the calculation method will be derived in Chapter 4. In Chapter 5 the calculation results are compared with the results of artificial traffic trials performed on a digital computer.

1. THE PRINCIPLE OF THE CALCULATION METHOD
The call congestion is divided into two parts [4, 7]:
1. The call congestion, caused by inlet blocking of the 1st stage.
2. The call congestion, caused by route blocking, i.e. the part of the call congestion, caused either by the limited access to the considered outgoing group or - in case of full access - by the state "all lines of this group busy".
In the following this idea will be applied also to systems with internal and external traffic. The various parts of the call congestion are calculated by means of the following probabilities of state:
- \( w(x) \) probability, that \( x \) outlets of the considered multiple of the 1st stage are occupied.
- \( p_j(x) \) probability, that \( x \) lines in the considered outgoing group No. \( j \) are busy.
These functions \( w(x) \) and \( p_j(x) \) are assumed to be independent of each other. The traffic \( X \) carried per multiple of the 1st stage as well as the traffic \( Y_j \) carried on the outgoing group No. \( j \) are prescribed. Furthermore, as an approximation, the known functions for the probabilities of state for full access groups with internal and external traffic are applied [1, 2]. Thus, the probability distributions \( w(x) \) and \( p_j(x) \) can be calculated iteratively by means of generating offered traffics \( A_0 \) and \( A_{0j} \) in such a way that the prescribed carried traffics \( X \) and \( Y_j \) result [2, 7].

2. DEFINITION OF "EXTERNAL" AND "INTERNAL" TRAFFIC

\[
\begin{array}{c}
\text{Stage} \\
\text{No. 1} \\
\hline
\text{external} \\
\text{internal (a)} \\
\text{internal (b)} \\
\text{Fig. 1}
\end{array}
\]

External traffic: A call establishes one path through the link system.

Internal traffic: One call establishes two paths through the link system. Hereby two cases have to be distinguished:

a) The origin and the destination of the internal connection are situated on the same multiple of the 1st stage (cf. Fig. 1, path (a)).

b) The origin and the destination of the internal connection are situated on different multiples of the 1st stage (cf. Fig. 1, path (b)).

To calculate the probability distribution for a trunk group - i.e. either for a multiple in the 1st stage or for an outgoing group behind the last
stage - the total internal traffic has to be di-

vided into two parts:

- that part which requires two paths in the con-
sidered group is named internal traf-

fic with regard to this group (cf. e.g. fig. 1; 
the connection (a) occupies two paths within 
the k1 outlets of the considered multiple).

- that part which establishes only one path in a 
considered group is named external traffic with 
regard to this group (cf. e.g. fig. 1; the connection 
(b) occupies one path only within the k1 outlets of the 
considered multiple).

3. THE OPERATION MODES

3.1 Operation Mode 1

stage 2

\[
\begin{array}{cccccc}
\text{group No. 1:} & y_1 & s & k_1 & n_{s1} \\
\text{group No. R:} & y_R & s & k_R & n_{sR} \\
\end{array}
\]

\[
\text{external traffic} \quad \text{incoming} \quad \text{internal traffic}
\]

\text{Fig. 2}

Each outlet of the link system in fig. 2 (right
hand side) can be used for traffic in outgoing or 
incoming direction. Therefore, internal traffic
needs two lines of the same outgoing group.

The parameters \(k_1, k_2, \ldots, k_R\) etc. of the 
structure of the link system are given (cf. fig. 
2). Let the carried traffics of a considered out-
grouping group No. R (\(j = 1, 2, \ldots, R\)) be prescribed, where:

\[
\begin{align*}
Y_{aj} & = \text{total carried traffic,} \\
i Y_{aj} & = \text{internal carried traffic,} \\
e Y_{aj} & = \text{external carried traffic,}
\end{align*}
\]

with

\[
Y_{aj} = Y_{aj} + e Y_{aj}
\]  

The ratio of internal to total carried traffic is:

\[
d_{aj} = \frac{Y_{aj}}{Y_{aj}}
\]

The total carried traffic of the link lines is:

\[
Y_{\text{total}} = \sum_{j=1}^{R} Y_{aj}
\]

Furthermore, the carried traffic of a multiple in
stage No. \(v\) (\(v = 1, 2, \ldots, s\)) amounts to:

\[
Y_v = \frac{Y_{\text{total}}}{K_v}
\]

The total internal carried traffic of the link
system is given by:

\[
i Y_{\text{total}} = \sum_{j=1}^{R} Y_{aj}
\]

With this total internal traffic of the link sys-
tem we find the share per multiple in stage No. 1:

\[
i Y_v = \frac{Y_{\text{total}}}{k_v}
\]

We assume that the internal traffic is split into
the \(e\) multiples of the 1st stage symmetrically.
To calculate the probability distribution for the 
outlets of the considered multiple in the 2nd stage
we have to calculate its own internal traffic. This internal traffic per multiple amounts to (cf. Chapter 2):

\[
\begin{align*}
Y_{j1} = \frac{Y_{j1}}{s_{j1}} = \frac{1}{\sum_{j=1}^{s_{j1}} Y_{j1}}
\end{align*}
\]

The external carried traffic per multiple is:

\[
e Y_{j1} = Y_{j1} - i Y_{j1}
\]

The ratio of internal to total carried traffic
per multiple of the 1st stage is:

\[
d_{j1} = \frac{i Y_{j1}}{Y_{j1}}
\]

3.2 Operation Mode 2

One of the two outgoing groups carries internal
traffic only, the other group carries external
traffic only (cf. fig. 3).

\text{Stage 2}

\[
\text{group No. 1:} & \quad Y_{j1} \\
\text{group No. 2:} & \quad Y_{j2}
\]

\[
\text{external traffic} \quad \text{incoming} \quad \text{internal traffic}
\]

\text{Fig. 3}

Link systems having this operation mode can be
found similarly in many telephone exchanges.

With given parameters of the structure as well
as the prescribed carried traffics on the outgoing
groups \(Y_{j1} = Y_{j1}\) and \(Y_{j2} = Y_{j2}\) we get:

\[
\begin{align*}
d_{j1} &= 1 \quad (\text{cf. eq. } (2)) \\
d_{j2} &= 0
\end{align*}
\]

Furthermore, we calculate \(Y_{j1}\) according to eq. (4), \(Y_{j2}\) according to eq. (7), \(i Y_{j1}\) according to
eq. (8), and \(i Y_{j2}\) according to eq. (9).

3.3 Operation Mode 3

\text{Stage 2}

\[
\text{group No. 1:} & \quad Y_{j1} \\
\text{group No. 2:} & \quad Y_{j2}
\]

\[
\text{external traffic} \quad \text{incoming} \quad \text{internal traffic}
\]

\text{Fig. 4}

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Just as in Chapter 3.2 we have only two outgoing groups. Group No. 1 contains internal as well as outgoing internal traffic, whereas, group No. 2 contains external as well as incoming internal traffic (cf. fig. 4).

Link systems having this operation mode are also applied in many switching systems.

The parameters of the structure and the carried traffic \(Y_x\) and \(Y_s\) on the outgoing groups are given.

Since an internal connection establishes only one line in an outgoing group, we get (cf. Chapter 2):

\[
d_{st} = d_{st} = 0
\]  

(11)

In this case the ratio \(d_{st}\) of internal to total carried traffic on the \(g_{st}\) link lines between stage No. 1 and stage No. 2 must be known. Thus it follows that (cf. Chapter 3.1, assumption for eq.(7)):

\[
d_{st} = \frac{d_{st}}{K_s} \tag{12}
\]

Furthermore we find the total carried traffic \(Y_v\) per multiple of stage No. \(v\) (\(v = 1, 2, \ldots, s\)) according to eq.(4):

\[
Y_v = d_{st} Y_s \tag{13}
\]

The external traffic \(Y_s\) follows from eq.(8).

4. CALCULATION OF CALL CONGESTION AND OFFERED TRAFFICS

4.1 Assumptions

- The probabilities of state of the outlets of each multiple of the 1st stage and those of the outgoing trunk groups are independent of each other.

- An internal call of the considered link system can be connected - between its outgoing and incoming line of stage No. \(s\) (last stage) - via further selector stages of the exchange.

- Time delays and losses caused by these selector stages are neglected.

- The external traffic is produced either by sources which are connected directly to the link system (outgoing external traffic) or by calls which come from the outside (incoming external traffic).

The calculation method makes no difference between outgoing and incoming external traffic. The sum of both external traffic is handled as a Poisson traffic offered to the left hand side of the system (cf. fig. 4).

4.2 Mean Accessibility \(m_j\)

The calculation of route blocking makes use of the Mean Accessibility \(m_j\) from an inlet of the 1st stage to the considered outgoing group No. 1. I.e. the quantity \(m_j\) can be interpreted as the average number of lines in the considered group No. 1 which can be hunted "free" or "busy" by the inlets of any multiple in stage No. 1 [4,5].

According to [4] we get:

\[
m_j = \left( \frac{K_j}{Y_j} \right) Y_j + \frac{d_j}{K_j} \tag{14}
\]

where

\(Y_j\) = ratio of the carried traffic on the outgoing group No. 1 to the total carried traffic of the system.

The following limitations have to be regarded for eq.(14):

\[
\sum_{Y_j} \left( \frac{K_j}{Y_j} - Y_j \right) \leq \varepsilon_{Y_j}, \quad Y_j = 1, 2, \ldots, s
\]

and

\[
m_j \leq n_j
\]

(15)

Example: If \(\sum_{Y_j} \left( \frac{K_j}{Y_j} - Y_j \right) > \varepsilon_{Y_j}\) and \(m_j > n_j\) we set

\[
\sum_{Y_j} \left( \frac{K_j}{Y_j} - Y_j \right) = \varepsilon_{Y_j} \quad \text{and} \quad m_j = n_j
\]

4.3 Congestion Probability \(G_j(x)\)

Route blocking \(b_{ij}\) is calculated as in the case of a one stage arrangement with constant limited accessibility \(m\). Outgoing groups with external traffic only are calculated by means of the MFF - formula [5,6]. For groups with internal and external traffic an adapted version of the MFF-formula is applied [2].

For a constant and integer accessibility \(k\) and for the number \(n\) of lines \(G(x)\) holds:

\[
G(x) = \frac{x}{y} \left( \frac{y}{x} \right)^{n-1}
\]

In our case, however, the Mean Accessibility \(m_j\) is as a rule not an integer value. Therefore, we calculate \(G(x)\) by means of a linear interpolation between the two neighbouring integer values of \(m_j\), \(m_{j-1}\) and \(m_{j+1}\), [3], where

\(m_{j-1} = \text{neighbouring integer value} \leq m_j\);

\(m_{j+1} = \text{neighbouring integer value} \geq m_j\).

We get

\[
G_j(x) = \frac{m_{j+1} - m_j}{m_{j+1}} \left[ \frac{m_j}{m_{j+1}} \right] G_j(x) + \frac{m_{j+1} - m_j}{m_{j-1}} \left[ \frac{m_{j+1}}{m_{j+1}} \right] G_j(x) \tag{16}
\]

4.4 Probabilities of State

4.4.1 The Probability of State \(p_j(x)\) for the Outgoing Group No. 1

Analogously to [1,2] we get the recurrence formula:

\[
p_j(x+2) = \frac{p_{A_{osj}}}{x+2} \cdot p_j(x+1) + \frac{2 p_{A_{osj}}}{x+2} \cdot p_j(x) + \frac{d_{s1}}{x+1} \cdot p_j(x) = 1 \tag{17}
\]

where

\(p_{A_{osj}} = \text{generating external traffic offered;}

\(A_{osj} = \text{generating internal traffic offered.}

Whereas the carried traffic \(Y_x\) and \(Y_s\) are prescribed, the quantities \(A_{osj}\) and \(A_{osj}\) have to be determined (by iteration) such that the following equations (18),(19), and (20) are fulfilled:

\[
x_{1} = e^{A_{osj}} \left( 1 - p_{j}(n_{1}) \right) \tag{18}
\]

\[
x_{2} = 2 \cdot A_{osj} \left[ 1 - \left( p_{j}(n_{2}) + p_{j}(n_{2}) - 1 \right) \right] \tag{19}
\]

Remark to eq.(19): The factor 2 depends on the fact that the internal connection establishes two paths in the considered outgoing group No. 1. Internal call congestion arises, if \(n_j(n_j - 1)\) lines are busy. Therefore, we get \(p_j(n_{1}) + p_j(n_{2}) = 1\) (cf. Chapter 4.3.1, eq.(23)).

Next, we calculate the ratio of internal to total traffic offered. It is defined by

\[
cos_j = \frac{1}{e^{A_{osj}} + A_{osj}} \tag{20a}
\]

\[
cos_j = \frac{d_{s1}}{2(1-d_{s1})} \left( 1 - p_{j}(n_{1}) - p_{j}(n_{2}) \right) \tag{20b}
\]
4.4.2 The Probability of State \( w(x) \) for a Multi-
pile of Stage No. 1

Analogously to eq.(17), (18), (19), and (20) we get:

\[
\begin{align*}
\bar{w}(x+2) &= \frac{A_{ot}}{x+2} \bar{w}(x+1) + \frac{2 \cdot A_{ot}}{x+2} \bar{w}(x) \\
\end{align*}
\]  

(21)

\[
\begin{align*}
\bar{d}_1 &= A_{ot} \cdot (1 - \bar{w}(k_1)) \\
\end{align*}
\]  

(22)

\[
\begin{align*}
\bar{d}_1 = 2 \cdot \beta_{ot} \cdot (1 - \bar{w}(k_1) - \bar{w}(k_1-1)) \\
\end{align*}
\]  

(23)

\[
\begin{align*}
c_{ot} &= \frac{\bar{d}_1}{\bar{d}_2 + 2(1 - \bar{d}_2)} \\
\end{align*}
\]  

(24)

where:

- \( \bar{d}_1 \) generating external traffic offered,
- \( \beta_{ot} \) generating internal traffic offered,
- \( c_{ot} \) ratio of internal to total offered traffic.

The probabilities of state \( w(x) \) can be calculated iteratively in the same way as \( p_j(x) \) in Chapter 4.1.

4.5 Inlet Blocking

The following equations exist for the different parts of inlet blocking:

- **External Inlet blocking**, referred to the external traffic offered:
  \[
  \beta_1 = \bar{w}(k_1) \\
  \]  

(25)

(Loss occurs if all \( k \), outlets of the considered multiple are busy.)

- **Outgoing Internal Inlet blocking**, referred to the internal traffic offered:
  \[
  \beta_2 = \beta_1 + \bar{w}(k_1) \\
  \]  

(26)

(Internal outgoing loss occurs also if all \( k \), outlets are busy.)

- **Incoming Internal Inlet blocking**, referred to the internal traffic offered:
  \[
  \beta_3 = \bar{w}(k_1-1) \\
  \]  

(27)

(In the state \( k_1-1 \) an outgoing internal call occupies the last free outlet, i.e. the incoming part of the internal call is lost.)

- **Total Internal Inlet blocking**, referred to the internal traffic offered:
  \[
  \beta_4 = \beta_1 + \beta_2 + \beta_3 \\
  \]  

(28)

(Outgoing internal blocking and incoming internal blocking are mutually exclusive events. Therefore, the two probabilities can be added.)

4.6 Route Blocking

4.6.1 Operation Mode 1

The following equations exist for the different parts of the route blocking:

- **External route blocking**, referred to the external traffic offered:
  \[
  \beta_1 = (m_{j_0} - m_{j_1}) \cdot p_{j_0}(m_{j_0}) \cdot \gamma_{j_0}(m_{j_0}) + \sum_{x \times \eta} p(x) \cdot \gamma_{j_1}(x) \\
  \]  

(29)

Remark to eq.(29): The call congestion of a trunk group with limited access and external traffic only is given by:

\[
\begin{align*}
\hat{b} = \sum_{x \times k} p(x) \cdot \hat{\gamma}_{j}(x) \\
\end{align*}
\]

In our case the \( \text{Mean Accessibility} \ m_j \) is not an integer. Therefore, we calculate \( \hat{b} \) by linear interpolation (cf. Chapter 4.3). After some transformations we get eq.(29).

- **Outgoing Internal route blocking**, referred to the internal traffic offered:
  \[
  \beta_2 = \beta_2 \\
  \]  

(30)

- **Incoming Internal route blocking**, referred to the internal traffic offered:
  \[
  \beta_3 = \sum_{x \times \eta} p(x) \cdot \gamma_{j_0}(x) \cdot \hat{\gamma}_{j_1}(x + 1) \\
  \]  

(31)

Remark to eq.(31): The call congestion of a trunk group with limited access and internal traffic only is given by [2]:

\[
\begin{align*}
\hat{b} = \sum_{x \times k} p(x) \cdot (1 - \gamma_{j_0}(x) \cdot \hat{\gamma}_{j_1}(x + 1) \\
\end{align*}
\]

Because \( q_1 \) is not an integer, we have also to interpolate linearly (cf. remark to eq.(29)).

- **Total internal route blocking**, referred to the internal traffic offered:
  \[
  \beta_4 = \beta_2 + \beta_3 \\
  \]  

(32)

4.6.2 Operation Mode 2

\( d_1 = 1 \) and \( d_2 = 0 \) are given (cf. Chapter 3.2).

With regard to \( d_1 = 1 \) we get \( \beta_2 = 0 \). The internal route blocking of the group No. 1 can be calculated according to eq.(30), (31), and (32).

With regard to \( d_1 = 0 \) we get \( \beta_2 = 0 \). The external route blocking of the outgoing group No. 2 can be calculated according to eq.(29).

4.6.3 Operation Mode 3

With regard to \( d_1 = d_2 = 0 \) (cf. Chapter 3.3) we get \( \beta_4 = 0 \). The external route blocking of the outgoing group No. 1 and No. 2 can be calculated according to eq.(29).

4.7 Call Congestions, Resulting from Inlet and Route Blocking

4.7.1 Call Congestion of External and Outgoing Internal Calls

The external call congestion, referred to the external traffic offered to group No. 1 is:

\[
\begin{align*}
\beta_{j_1} &= \beta_1 + (1 - \beta_1) \beta_2 \\
\end{align*}
\]  

(33)

(\( \beta_2 \) and \( \beta_{j_2} \) are assumed to be independent.)

The outgoing internal call congestion, referred to the internal traffic offered is:

\[
\begin{align*}
\beta_{j_2} &= \beta_4 \\
\end{align*}
\]  

(34)

4.7.2 Call Congestion of Incoming Internal Calls

4.7.2.1 General Remarks

The incoming internal call congestion, referred to the internal traffic offered, has to be calculated as follows:

We have to distinguish between 4 cases of blocking (cf. fig. 5):

\[
\begin{align*}
\begin{tikzpicture}
\draw [->, thick] (0,0) -- (3,0) node[above] {outgoing internal connection};
\draw [->, thick] (3,0) -- (6,0) node[above] {incoming internal connection};
\end{tikzpicture}
\end{align*}
\]
Case α) The destination multiple is identical to the origin multiple; the incoming internal call cannot find an idle link path from stage No. s to the destination multiple in stage No. 1.

Case β) The destination multiple is identical to the origin multiple; all free lines of the incoming group are connected with multiples of the stage No. s, whose paths to the destination multiple are blocked.

Case γ) The destination multiple is not identical to the origin multiple; the incoming internal call cannot find an idle link path from stage No. s to the destination multiple in stage No. 1.

Case δ) The destination multiple is not identical to the origin multiple; all free lines of the incoming group are connected with multiples of the stage No. s, whose paths to the destination multiple are blocked.

For each of the 3 operation modes these 4 cases have to be considered.

4.7.2.2 Operation Mode 1

Case α)
 biochemical \( k_{\text{c}(ij)} = \frac{1}{\varepsilon_{i}} \cdot k_{b_{i}} (1 - \varepsilon_{b_{s}}) \), (35)

where
\( \frac{1}{\varepsilon_{i}} \) = probability (destination multiple identical to origin multiple),
\( 1 - \varepsilon_{b_{s}} \) = prob. (no outgoing route blocked),
\( k_{b{i}} \) = prob. (destination multiple blocked).

Case β)
 biochemical \( \delta _{\text{c}(ij)} = \frac{1}{\varepsilon_{i}} \cdot k_{b_{i}} (1 - \varepsilon_{b_{i}}) \), (36)

where
\( \frac{1}{\varepsilon_{i}} \) as in case α),
\( 1 - \varepsilon_{b_{i}} \) = prob. (destination multiple not blocked),
\( k_{b_{i}} \) = prob. (incoming route has no idle path to the destination multiple).

Case γ)
 biochemical \( \delta _{\text{c}(ij)} = (1 - \frac{1}{\varepsilon_{i}}) \cdot k_{b_{i}} (1 - \varepsilon_{b_{i}}) (1 - \varepsilon_{b_{s}}) \), (37)

where
\( 1 - \frac{1}{\varepsilon_{i}} \) as in case γ),
\( 1 - \varepsilon_{b_{i}} \) = prob. (destination multiple not identical to origin multiple),
\( 1 - \varepsilon_{b_{s}} \) = prob. (origin multiple not blocked),
\( k_{b_{i}} \) as in case α),
\( k_{b_{s}} \) = prob. (destination multiple blocked).

Case δ)
 biochemical \( \delta _{\text{c}(ij)} = (1 - \frac{1}{\varepsilon_{i}}) \cdot k_{b_{i}} (1 - \varepsilon_{b_{i}})^{2} \), (38)

where
\( 1 - \frac{1}{\varepsilon_{i}} \) as in case γ),
\( 1 - \varepsilon_{b_{i}} \) = prob. (origin and destination multiple not blocked),
\( k_{b_{i}} \) as in case β).

With eq. (36), (37), and (38) the total call congestion for incoming internal calls holds:
 biochemical \( k_{\text{c}(ij)} = k_{\text{c}(ij)} + \delta _{\text{c}(ij)} + \delta _{\text{c}(ij)} + \delta _{\text{c}(ij)} \), (39)

4.7.2.3 Operation Mode 2

For group No. 1 (internal traffic only) we find \( k_{\text{c}(ij)} = 0 \). Therefore, only eq. (35), (36), (37), (38), and (39) are relevant.

In group No. 2 we have external traffic only, i.e. \( k_{\text{c}(ij)} = 0 \). Equation (39) is valid.

4.7.2.4 Operation Mode 3

The internal traffic is offered to group No. 1 only (cf. Chapter 3.3), i.e. \( j_{\text{c}(ij)} = 0 \).

The 4 parts of the call congestion for incoming internal calls are:

Case α) \( k_{\text{c}(ij)} \) according to eq. (35)

Case β) \( \delta _{\text{c}(ij)} = \frac{1}{\varepsilon_{i}} \cdot k_{b_{i}} (1 - \varepsilon_{b_{i}}) (1 - \varepsilon_{b_{s}}) \), (40)

where
\( 1 - \varepsilon_{b_{i}} \) = prob. (no outgoing route blocked),
\( 1 - \varepsilon_{b_{s}} \) = prob. (incoming route has no idle path to the destination multiple),
\( 1/\varepsilon_{i} \) and \( 1 - \varepsilon_{b_{i}} \) as in eq. (36).

Case γ) \( \delta _{\text{c}(ij)} \) according to eq. (37)

Case δ) \( \delta _{\text{c}(ij)} = (1 - \frac{1}{\varepsilon_{i}}) \cdot k_{b_{i}} (1 - \varepsilon_{b_{i}})^{2} (1 - \varepsilon_{b_{s}}) \), (41)

(cf. eq. (38) and (40), respectively.)

The total call congestion \( k_{\text{c}(ij)} \) is calculated according to eq. (39).

4.7.3 Call Congestion for Internal Traffic

Generally, we get the total internal call congestion, referred to the internal traffic offered from:
 biochemical \( k_{\text{c}(ij)} = k_{\text{c}(ij)} + k_{\text{c}(ij)} \), (42)

4.8 Offered Traffic

4.8.1 Operation Modes 1 and 2

External Traffic offered is:
 biochemical \( A_{s} = \frac{e_{s}}{1 - e_{s}} \), (43)

Internal Traffic offered is:
 biochemical \( A_{s} = \frac{Y_{s}}{Z (1 - Y_{s})} \), (44)

From eq. (43) and (44) we obtain the total traffic \( A_{s} \) offered to the group No. 1:
 biochemical \( A_{s} = \frac{A_{s} + A_{s}}{A_{s}} \), (45)

The ratio of internal to total traffic offered becomes:
 biochemical \( A_{s} = \frac{A_{s}}{A_{s}} \), (46)

4.8.2 Operation Mode 3

The prescribed traffic parameters \( Y_{t1}, Y_{t2} \) and \( d_{t1} \) (cf. Chapter 3.3) yield the total traffic \( Y_{ttotal} = Y_{t1} + Y_{t2} \), according to eq. (3) and its internal share:
 biochemical \( Y_{ttotal} = d_{t1} \), (47)

This internal share of the total traffic carried is divided equally into the outgoing direction No. 1 and the incoming direction No. 2:
 biochemical \( Y_{s1} = Y_{s2} = \frac{Y_{ttotal}}{2} \), (48)

(Reform to index: In the operation mode 3 internal calls occupy one line per outgoing and incoming group, respectively.)

Outgoing Group No. 1:

We obtain:
 biochemical \( A_{s1} = \frac{Y_{s1}}{1 - e_{s1}} \), (49)
and
\[ A_{st}^* = \frac{Y_{st}}{1 - e^{b(1)}} \]  \hspace{1cm} (50)

Furthermore,
\[ A_{st} = eA_{st}^* + A_{st}^* \]  \hspace{1cm} (51)

and
\[ c_{st} = A_{st} \]  \hspace{1cm} (52)

Outgoing group No. 2:
We get
\[ A_{s2} = eA_{s2} = \frac{Y_{s2} - Y_{s2}^*}{1 - e^{b(2)}} \]  \hspace{1cm} (53)

and
\[ c_{s2} = 0 \]  \hspace{1cm} (54)

4.0 Call Congestions, Referred to the Total Traffic Offered

4.1 Operation Modes 1 and 2
We obtain:
External call congestion:
\[ eB_j = (1 - c_{sj})e^b(k) \]  \hspace{1cm} (55)

Internal call congestion:
\[ B_j = c_{sj}e^b(k) \]  \hspace{1cm} (56)

Total call congestion:
\[ B_j = eB_j + B_j \]  \hspace{1cm} (57)

4.2 Operation Mode 3
Outgoing group No. 1:
In eq.\((55),(56),(57)\) the traffic ratio \(e_{st}\) (according to eq.\((46)\)) has to be replaced by \(e_{st}\) (according to eq.\((52)\)).

Outgoing group No. 2:
Since \(c_{s2} = 0\) (cf. eq.\((54)\)) we obtain:
\[ B_2 = eB_2 = e^{b(2)} \]  \hspace{1cm} (58)

4.10 Mean Call Congestion, with Respect to All Outgoing Groups
We obtain:
\[ B_{total} = \frac{\sum A_jB_j}{\sum j=1 A_j} \]  \hspace{1cm} (59)

5. COMPARISON BETWEEN CALCULATION AND ARTIFICIAL TRAFFIC TRIALS

In all diagrams \(\%\) means the test results with a confidence interval of 95%. The solid lines are obtained with the presented calculation method.

5.1 Two-Stage Link System with Operation Mode 1

Diagram 1: Total call congestion \(B_{total}\) as function of the carried traffic per outlet \(Y_{st}/n_{st}\), parameter \(d_{s1}\).
1: \(d_{s1} = 1\), internal traffic only
2: \(d_{s1} = 0.7\), mixed internal and external traffic
3: \(d_{s1} = 0\), external traffic only.

Diagram 2: 1 : Internal call congestion, referred to the internal traffic offered as function of \(Y_{s1}/n_{s1}\) \((d_{s1} = 0.5)\).
2 : External call congestion, referred to the external traffic offered as function of \(Y_{s1}/n_{s1}\) \((d_{s1} = 0.5)\).

Diagram 3: 1 : Internal call congestion, referred to the internal traffic offered as function of the carried traffic per outlet \(Y_{s1}/n_{s1} = Y_{s2}/n_{s2} = Y_{s3}/n_{s3} \) \((d_{s1} = 1)\).
2 : External call congestion, referred to the external traffic offered as function of \(Y_{s2}/n_{s}(d_{s2} = 0)\).
5.3 Four-Stage Link System with Operation Mode 3

Diagram 4: Total call congestion as function of the carried traffic per outlet \( I_b / n_x = Y_x / n_x = Y_s / n_s \), parameter \( d_x \).

1: \( d_x = 0.5 \), mixed internal and external traffic.
2: \( d_x = 0 \), external traffic only.

Diagram 5: 1: \( b_{(1)} \)
2: \( e_{(2)} \)

structure of the link system: cf. diagc 4

REFERENCES

[1] Håmmblom, N.: Traffic loss of a circuit group consisting of both-way circuits which is accessible for the internal and external traffic of a subscriber group. Tele 1959, pp. 79-92

b) A.B.U. 22(1968), Heft 3

Institute for Switching and Data Techniques, University of Stuttgart, Monograph 1966

[4] Lotze, A.: Computation of Time- and Call-Congestion in link systems with two and more selector-stages and with preselection or group-selection according to an approximation method, which is named "Combined Inlet- and Route-Blocking". Institute for Switching and Data Techniques, University of Stuttgart, Proceedings Nr. 3 (1963)

Institute for Switching and Data Techniques, University of Stuttgart, 1962

NTZ 14 (1961), H.9, p.241

3. ITC New York 1967, prebook of the congress, pp.242-251